Land reclamations of dredged mud; consolidation of soft soils
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1. **Introduction**

Generally, mud dredged from harbour docks and navigation channels in estuaries is dumped at nearby dumping grounds within the system. Dredged mud can also be dumped far away (at sea) from the dredging locations. However, this may be expensive if the sailing distance is relatively large.

Another option may be the construction of land reclamations (new land for nature or for industrial purposes, *Figure 1.1*) in the form of coastal extensions and/or small islands consisting of compartments (50 to 100 ha) surrounded by low sandy/rocky dikes or sheet pile walls and filled with dredged mud. New land for industrial purposes with heavy loads requires the use of high-density mud. In the case of extreme loads (airport landing strips) it may be necessary to use cement-type binders for soil improvement (Japan).

An additional advantage of a land reclamation of mud is the removal of mud from the system resulting in less turbid water and less dredging activities on the long term.

Essential for a good solution is an adequate design in combination with in-situ monitoring, focussing on the composition, rheology and density of the mud to be dredged (and pollution/turbidity levels) and the mud to be dumped (in place and time).

This document presents information on:
- filling of compartments,
- crust formation, cover layer and drainage,
- consolidation of soft fluid mud to firm soil,
- mud pollution.

![Mud land reclamation and mud island](image1)

*Figure 1.1  Mud land reclamation and mud island*

The outer protection (enclosure dike/dam) of the land reclamation can be made of:
- steep-sloped sand body protected by stones (0.2 to 0.3 m);
- mild-sloped sand body with a beach (slope of 1 to 25 for 0.2-0.3 mm sand) in mild wave conditions.

*Figure 1.2* shows an example of a sand dike with a mild sloping beach.

![Sand dike/beach](image2)

*Figure 1.2  Sand dike/beach*
2. Dredging of mud from docks and channels for new land

2.1 Mud properties

It is very efficient to use mud trapped in docks and channels as source material for land reclamation works. Most basic for the complete cycle of dredging, filling and consolidation methods to be used are the mud properties.

The most important mud properties to be determined by field and laboratory work, are:

- in-situ wet bulk density over the depth of the deposits (bulk density profile);
- fraction clay/lutum < 4 μm; fraction silt 4 to 63 μm; fraction fine sand > 63 μm; content organic materials; content shell;
- settling velocity as function of concentration;
- consolidation parameters;
- critical shear stress for erosion;
- yield stress and flow point stress.

Wet and dry bulk density

Mixtures of mud and sand have varying dry density values depending on various parameters (percentage mud < 63 μm, percentage clay < 8 μm, layer thickness, degree of consolidation/packing).

Figure 2.1 shows various types of packing (Wu and Li, 2017).

The dry density can also be expressed in terms of the porosity, which is a measure of the void space in the sediment mixture. Porosity (η) is defined as the volume of voids per unit volume of the mixture, as follows: η = V_v/(V_v + V_s) with: V_v = volume of voids, V_s = volume of solids.

The porosity is related to the dry density of the mixture: ρ_{dry,mixture} = ρ_s(1-η), with ρ_s = sediment density.

Separate packing of sand and mud (Figure 2.1 left) can be described by:

\[
\frac{V_{v,sand} + V_{v,mud} + V_{mud} + V_{sand}}{V_{sand}/(1-\eta_{sand}) + V_{mud}/(1-\eta_{mud})} = \frac{V_{sand} + V_{mud}}{(1-\eta)}
\]

\[
\frac{V_{sand}/(1-\eta_{sand})}{(1-\eta)} + \frac{V_{mud}/(1-\eta_{mud})}{(1-\eta)} = \frac{1}{(1-\eta)}
\]

\[
\rho_{sand}/(1-\eta_{sand}) + \rho_{mud}/(1-\eta_{mud}) = 1/(1-\eta) \text{ or more generally } \sum_{i}^{n} \rho_i/(1-\eta_i) = 1/(1-\eta)
\]

\[
\rho_{sand}/\rho_{dry,sand} + \rho_{mud}/\rho_{dry,mud} = 1/\rho_{dry,mixture}
\]
Land reclamations of mud

Note: Land reclamations of mud

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with: \( p_{\text{sand}} = \) percentage sand, \( p_{\text{mud}} = \) percentage mud, \( p_{\text{sand}} + p_{\text{mud}} = 1 \), \( p= \) percentage of fraction \( i \), \( n_{\text{sand}} = \) porosity of fraction \( i \), \( n_{\text{mud}} = \) porosity of mud fraction.

However, separate packing without filling of the voids of the sand fraction by the fine fraction is not realistic. Wu and Li (2017) have proposed a model including the filling of the voids of the coarse sand fraction. Two extreme cases are possible: 1) coarse packing with all sand particles in contact with each other and the voids of the sand fraction filled with fines (Figure 2.1 middle) and 2) fine packing when the coarse sand particles are not in contact but dispersed within the fine fraction (Figure 2.1 right).

Wu and Li (2017) have proposed a filling coefficient \( \beta \), which is the voids portion of the sand fraction filled up by fines. If the material density of mud and sand fraction is the same, the following expression can be derived:

\[
\frac{p_{\text{sand}}}{(1-\eta_{\text{sand}})} (1-\beta) + \beta (1-\eta_{\text{sand}}) + \frac{p_{\text{mud}}}{(1-\eta_{\text{mud}})} = 1/(1-\eta)
\]

\[
\frac{p_{\text{sand}}}{(1-\eta_{\text{sand}})} (1-\beta) + \beta (1-\eta_{\text{sand}}) + \frac{p_{\text{sand}}}{(1-\eta_{\text{sand}})} + \frac{p_{\text{mud}}}{(1-\eta_{\text{mud}})} = 1/(1-\eta)
\]

\[
\frac{p_{\text{sand}}}{\rho_{\text{dry,sand}}} (1-\beta) + \beta \rho_{\text{dry,sand}} + \frac{p_{\text{mud}}}{\rho_{\text{dry,mud}}} = 1/\rho_{\text{dry,mixture}}
\]

with: \( \beta = \) coefficient (0 to 1). Wu and Li (2017) have proposed predictive expressions for the \( \beta = \) coefficient. If measured data are available, the dry mud density can be determined from:

\( p_{\text{sand}} \rho_{\text{dry,sand}} + p_{\text{mud}} \rho_{\text{dry,mud}} = \rho_{\text{dry,mixture}} \)

Using this, the \( \beta \)-coefficient can also be determined.

Van Rijn and Barth (2018) have proposed empirical equations for the dry bulk density of mud and sand mixtures as function of percentage mud/sand and percentage clay (fraction < 8 \( \mu \)m).

Sediment composition

Three methods are available for size determination, as follows:

• sediment size using sieves (direct method);
• sediment size using Laser-Diffractron (LD) instrument (direct method);
• settling velocity of primary particles (deflocculated with peptiser) using Sedigraph instrument and sediment diameter based on Stokes settling formula (indirect method);

The LD-instrument (Malvern) is an attractive instrument to determine the particle size distribution (PSD) of the primary particles because the measurement is fast and simple using diluted samples. The MALVERN measures the volume of the particles which are converted to sphere-diameters. The laser beam passes through a dispersed particulate sample and the angular variation in intensity of the scattered light is measured. Large particles scatter light at small angles relative to the laser beam and small particles scatter light at large angles, see Figure 2.2. The particle size is reported as a volume equivalent sphere diameter.

Figure 2.2  Left: Light scattering at small and large angles; Laser-Diffraction method (Malvern)
Right: X-ray absorption of SEDIGRAPH instrument
The LD-instruments are known to underestimate the clay fraction (d < 2 µm). The fines are partly shadowed by the larger particles. This can be partly overcome by separating the finest fraction from the bulk by means of a settling test, in which the finest fraction remains in suspension after one hour. The ratio of the mass of the finest fraction to the overall bulk mixture can be calculated, and the PSD can be corrected with the PSD of the finest fraction.

The SEDIGRAPH instrument is based on X-ray absorption to determine the decrease of the sample sediment mass (concentration) as function of time from an initially uniform suspension. The SEDIGRAPH uses a narrow beam of X-rays to measure directly the particle concentration in the liquid medium, see Figure 2.2. This is done by first measuring the intensity of a baseline or reference X-ray beam which is projected through the cell windows and through the liquid medium prior to the introduction of the sample. A homogeneously dispersed mixture of solid sample (deflocculated using peptiser) and liquid is next pumped through the cell. The attenuated X-ray beam is measured to establish a value for full scale attenuation. Agitation of the mixture is ceased and the dispersion is allowed to settle while X-ray intensity is monitored. During the sedimentation process, the largest particles fall below the measuring level, and progressively finer and finer particles do so until only the finest remain near the top of the measuring cell. The settling velocity is converted to a PSD using Stokes’ law. This conversion introduces an error, since Stokes’ law is valid for spherical particles, whereas clays consist of plate-like particles which settle somewhat slower. Hence, the Sedigraph will underestimate the particle size in the silt range and overestimate the clay fraction.

Figure 2.3 shows an example of a sediment size distribution based on sieving for the sand particles > 63 µm and on the Sedigraph-instrument for the finer particles < 63 µm.

**Settling velocity**

The settling velocities of both the primary (non-flocculated) and flocculated aggregates are important and can be determined, as follows:
- primary particles using Sedigraph instrument;
- flocculated material using a settling test.

The settling tube/column is a perspex cylinder with internal diameter of 100 mm and height of about 0.5 m, see Figure 2.4. A small plastic tapping tube (hose with a clamp) is present at about 70 mm above the bottom of the column. A suspension of seawater and mud (volume of about 2.5 liter) can be prepared with an initial concentration in the range of 500 to 5000 mg/l. The suspension is mixed thoroughly (manually) using a simple wooden mixing stick before the start of the settling process. Small samples (about 100 ml) of water and mud are taken after 5, 60, 180, 300, 600, 1800, 3600 and 7200 seconds to determine the decreasing mud concentrations over time. Immediately after each sample withdrawal, the water surface
level of the settling column above the tap opening is measured. Most of the suspended mud has settled out to the bottom after 2 hours. The mud samples are filtrated (using glass fibre filter material with size of 0.45 \( \mu \)m; each filter is numbered and preweighed) and weighed to determine the mud concentration.

![Perspex settling column (internal diameter=100 mm) and filtration unit](image)

**Figure 2.4**  
Perspex settling column (internal diameter=100 mm) and filtration unit

**Figure 2.5** shows an example of measured settling velocities of flocculated and non-flocculated mud. The latter has been determined using the SEDIGRAPH-instrument after mixing the sample with peptiser (for defloccution).

![Settling velocities of flocculated and non-flocculated mud; Noordpolderzijl, Netherlands](image)

**Figure 2.5**  
Settling velocities of flocculated and non-flocculated mud; Noordpolderzijl, Netherlands

**Consolidation tests**  
Consolidation tests should be done in saline water (native seawater) with initial suspension concentrations of \( c_0 = 10, 30, 50, 100, 200 \) and \( 300 \) kg/m\(^3\). Each mixture is poured into a settling column (plastic cylinder /tube closed at bottom (see **Figure 2.6**) and stirred mechanically to create a homogeneous suspension of seawater and mud. After that, the settling starts and the position of the interface between the clear water and the suspension are recorded over time. The consolidation process consists of two clear phases: 1) flocculation+hindered settling phase and 2) consolidation phase.
2.2 Dredging methods

The most appropriate dredging method for muddy land reclamations depends on the available budget for construction works and the available time for consolidation of the new land. Hydraulic dredging methods based on the pumping of a mud slurry through a pipeline are relatively cheap, but the mud delivered at the reclamation site generally has a low density so that a relatively long consolidation time is required to obtain usable new land. Mechanical excavation of mud generally yields relatively high mud densities at the site, but the method is very time consuming and relatively expensive per unit volume of mud. Table 2.1 presents a summary of the available dredging methods.

<table>
<thead>
<tr>
<th>Type of dredger</th>
<th>Type of soils</th>
<th>Maximum water depth (m)</th>
<th>Production (m³/hour)</th>
<th>Transportation</th>
<th>Wet/dry bulk density at delivery (kg/m³)</th>
<th>Turbidity levels</th>
</tr>
</thead>
<tbody>
<tr>
<td>Clamshell Grab dredger (crane; backhoe)</td>
<td>Mud; Sand</td>
<td>Crane: 50 Backhoe: 20</td>
<td>100-500 (grab volume 5 to 10 m³)</td>
<td>Discontinuous; barges; conveyer belt</td>
<td>1400-1800; (600-1300) 70% mud 30% water</td>
<td>Medium (spill around barges)</td>
</tr>
<tr>
<td>Screw (Auger) dredger</td>
<td>Mud (&lt; 20% sand)</td>
<td>20</td>
<td>1000-3000 (head moves forward in broad lanes)</td>
<td>Continuous through pipeline</td>
<td>1300-1450; (450-700) 50% mud 50% water</td>
<td>Very low (almost nil with silt screens)</td>
</tr>
<tr>
<td>Cutter dredger</td>
<td>Mud; Sand</td>
<td>40</td>
<td>1000-5000 (horizontally swinging head)</td>
<td>Continuous through pipeline</td>
<td>1100-1300 (150-450); 30% mud 70% water</td>
<td>Low (almost nil with silt screens)</td>
</tr>
<tr>
<td>Trailing suction hopper dredger</td>
<td>Mud; Sand</td>
<td>30</td>
<td>1000-10000 (depending on sailing distance)</td>
<td>Discontinuous filling through pipeline</td>
<td>1100-1300 (150-450); 30% mud 70% water</td>
<td>Low-High (depending on overflow of fines)</td>
</tr>
</tbody>
</table>

Table 2.1 Dredging methods
**Dredging characteristics**

Mechanical grab dredging using a closed clamshell grab by crane or backhoe (Figures 2.10 and 2.11) delivers mud at the site with the highest bulk density, but the method is laborious and time consuming as barges are required to bring the high-density mud to the site. If the site is accessible over water, the barges can sail into the site and dump the mud through bottom doors. Otherwise, the barges are moored at a station outside the site and are unloaded by a grabcrane/backhoe again into another barge with bottom doors inside the site. Mud pollution/turbidity may be significant by spilled mud at the unloading site.

A cutter-suction dredger (see Figure 2.7) has a cutting mechanism (rotating screw and water jets) at the suction inlet. The cutting mechanism loosens the bed material and transports it to the suction mouth. The dredged material is usually sucked up by a wear-resistant centrifugal pump and discharged either through a pipe line or to a barge. Cutter-suction dredgers are most often used in areas with firm gravel/sand/clay layers or when a relatively deep sand mining pit has to be made.

A screw-Auger dredger operates like a cutter suction dredger, but the cutting tool is a rotating screw at right angles to the suction pipe (see Figures 2.7, 2.8). A horizontal hydraulic Auger dredger moves forward and dredges material away in broad lanes (dredge cuts), which are easy to track by echo-sounder. Self-propelled Auger dredgers are available (Figure 2.8) without the use of anchors or cables. Horizontal Auger heads (www.dopdredgepumps.com) can also be attached to a backhoe boom (Figure 2.8).

**Turbidity**

Cutter-suction dredgers generate a cloud of dredged material into the water, which is pumped/sucked into the mouth of the dredge pump. However, cutter-suction dredgers are not able to suck all that material up and may leave as much as 5% of all disturbed solids in the ambient water.

Horizontal hydraulic Auger dredgers push the dredged material into a shroud that directs the material into the pump’s suction mouth. The shrudging of material enables horizontal hydraulic Auger dredgers to suck up almost all materials. Silt screens can be used to reduce the spreading of spilled mud (Figure 2.9).

Grab dredging with open buckets/grab yields major mud spill during hoisting operations, Figure 2.10Upper. Low-turbidity closed clamshell grabs of 5 and 7 m³ are delivered by The Grab Specialist (www.tgs-grabs.nl).
Note: Land reclamations of mud
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Figure 2.9  Silt screens

Figure 2.10  Grab dredging
Upper: Open grabs with major mud spill during hoisting
Middle: Closed hydraulic clamshell grab (Grab Specialist; www.tgs-grabs.nl; Netherlands)
Lower: Closed mechanical clamshell grab for crane (2 cables); (www.tgs-grabs.nl)
2.3 Dumping/Unloading

Generally, the compartment to be filled with mud is fully enclosed by a ring dike so that the mud can be delivered at the site through a floating pipeline or by small barges with bottom doors moving in the compartment (water depth of 3 m required).

The unloading process can be subdivided into (Figure 2.12):

- hydraulic (pump and pipeline);
  - hoppers use their own equipment to pump the mud from the mooring station through a floating pipe line; pipe exit is moved around the compartment by a small boat for gradual filling;
  - cutters and augers deliver (pump) the mud through a floating pipeline directly to the site;
  - barges with dredged mud are unloaded by pump (+pipeline) attached to crane; Figure 2.12Upper;
- mechanical-hydraulic (Figure 2.12Middle);
  - barges with dredged mud are unloaded by a grab-crane/backhoe into a storage container and from that by a plunger pump (+ floating pipeline);
- mechanical (Figure 2.12Lower);
  - barges are unloaded by a crane filling a dumper truck or another barge at the inside of the compartment; the barge sails into the compartment to dump its load (split-hull or bottom doors).
Figure 2.12  Unloading methods;
Upper:  Auger screw pump (mechanical-hydraulic unloading);
Middle:  Soil plunger pump (mechanical-hydraulic unloading);
Lower:  Barges on both sides (fully mechanical unloading).
The highest bulk density (favourable for rapid consolidation) can be delivered at the site using a mechanical unloading system consisting of a grab operated by a crane filling another barge at the inside, which sails into the compartment and unloads by opening bottom doors (Figure 2.12 Lower). However, this method is relatively expensive as the complete cycle of dredging and dumping is mechanical.

A compromise may be the unloading of barges by using a special pump for high-density fluids-soils (Figure 2.12 Upper/Middle). The wet bulk density may go down by 5% to 10%, because some water may be required to reduce the pipeline friction.

Two types of pumps are available which can be used to pump fluids-solids with a wet bulk density of around 1500 kg/m$^3$ over distances up to 0.5 km:

- **vertical Auger-pump (Figures 2.12 and 13):** production rate of 300-500 m$^3$/hour; pressure head of about 35 m; pipeline of 0.2 m up to 500 m long; pump unit price of about 0.25 million Euro;
- **soil plunger pump (Figure 2.12 and 2.14):** production rate of 200-300 m$^3$/hour; pipeline of 0.2 m up to 500 m; pump should be placed at high position so that gravity helps the plunger/pushing process; pump unit price of about 0.35 million Euro.

The pipeline friction is given by (laminar flow): $f = \frac{64 \nu}{(v \cdot D)}$, with $\nu =$ kinematic viscosity coefficient, $v =$ mud velocity in pipe $= \frac{Q}{(0.25 \pi D^2)}$, $D =$ pipe diameter.

Using: $Q =$ 500 m$^3$/hour $\approx 0.15$ m$^3$/s; $\nu =$ 0.0001 m$^2$/s (100 times thicker than water), $D =$ 0.2 m yields: $v =$ 5 m/s and thus $f =$ 0.007.

The head loss per m length is given by: $\Delta H/L = \left[\frac{f}{D}\right] \left[\frac{v^2}{(2g)}\right] = \left[\frac{0.07}{0.2}\right] (25/20) = 0.03$ or 15 m over 500 m.

The Auger pump can deliver an hydraulic head of about 30 m for mud with a wet bulk density of 1500 kg/m$^3$.

Given these values, a production rate in the range of 300 to 500 m$^3$/hour is realistic.

A circular compartment with a volume of 1 million m$^3$ (diameter 350 m; height=10 m) can be filled in 2500 hours or 250 working days of 10 hours.

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**Figure 2.13** Auger screw pump; length= 3.5 m; width= 0.8 m; weight= 600 kg; operated by crane/backhoe (DOP150; Damen dredging equipment, Nijkerk, The Netherlands; www.dopdredgepumps.com)
Figure 2.14 Soil plunger pump (Degra pump MP series 300); capacity 200-300 m³/hour; pipeline 0.2 m (www.degra.info; www.werktuigen.nl)

<table>
<thead>
<tr>
<th>Process</th>
<th>Hopper dredging</th>
<th>Cutter-suction dredging</th>
<th>Auger suction dredging</th>
<th>Grab/backhoe dredging</th>
</tr>
</thead>
<tbody>
<tr>
<td>Production rate (m³/hour)</td>
<td>1000-10000</td>
<td>1000-3000</td>
<td>1000-3000</td>
<td>1000-2000</td>
</tr>
<tr>
<td>Wet/Dry bulk density at dredging site (kg/m³)</td>
<td>1100-1200 (150-300)</td>
<td>1200-1300 (300-450)</td>
<td>1100-1200 (150-300)</td>
<td>1300-1400 (450-600)</td>
</tr>
<tr>
<td>Transportation of mud</td>
<td>hydraulic (discontinu)</td>
<td>hydraulic (continu)</td>
<td>hydraulic (continu)</td>
<td>hydraulic (discontinu)</td>
</tr>
<tr>
<td>Hindrance to port activities</td>
<td>minimum</td>
<td>maximum</td>
<td>medium</td>
<td>maximum</td>
</tr>
<tr>
<td>Pollution (turbidity production)</td>
<td>minimum</td>
<td>maximum</td>
<td>medium</td>
<td>medium: open grab</td>
</tr>
<tr>
<td>Unloading</td>
<td>pump station close to land and pipeline</td>
<td>pump station close to land and pipeline</td>
<td>pipeline 1) grab+ barge/truck 2) Auger-pump and pipeline</td>
<td>pipeline 1) grab+ barge/truck 2) Auger-pump and pipeline</td>
</tr>
<tr>
<td>Wet/Dry bulk density (kg/m³) delivered at land site</td>
<td>1100-1200 (150-300)</td>
<td>1200-1300 (300-450)</td>
<td>1100-1200 (150-300)</td>
<td>1300-1400 (450-600)</td>
</tr>
<tr>
<td>Usable land of mud</td>
<td>after 10-15 years</td>
<td>after 10-15 years</td>
<td>after 10-15 years</td>
<td>after 5-10 years</td>
</tr>
<tr>
<td>Costs (Euro/m³)</td>
<td>5-10</td>
<td>5-10</td>
<td>5-10</td>
<td>5-10</td>
</tr>
</tbody>
</table>

Table 2.2 Detailed characteristics of dredging and dumping (unloading) methods
2.4 Bulking factor

It should be realized that the volume of soil to be excavated at the source site increases due to the dredging activities as the consolidated source material with a relatively high mud bulk density is dredged away from the source site and dumped at the building site with a lower bulk density. As the sediment mass remains the same, the continuity equation yields:

\[
V_{\text{source}} \rho_{\text{dry,source}} = V_{\text{site}} \rho_{\text{dry,site}} \\
V_{\text{source}} = (\rho_{\text{dry,source}} / \rho_{\text{dry,site}}) V_{\text{source}} = \left[ (\rho_{\text{wet,source}} - \rho_w) / (\rho_{\text{wet,site}} - \rho_w) \right] V_{\text{source}}
\]

with:

\[
f_{\text{bulk}} = (\rho_{\text{dry,source}} / \rho_{\text{dry,site}}) = \left[ (\rho_{\text{wet,source}} - \rho_w) / (\rho_{\text{wet,site}} - \rho_w) \right] = \text{bulking factor;}
\]

\[
\rho_{\text{wet}} = \rho_w + (1 - \rho_w / \rho_s) \rho_{\text{dry}} = \text{wet bulk density; } \rho_{\text{dry}} = \text{dry bulk density; } \rho_w = \text{water density; } \rho_s = \text{sediment density.}
\]

The bulking factor \( f_{\text{bulk}} \) depends on the type of soil and on the excavation method. Hydraulic dredging leads to relatively large bulking factors as added water is required to dredge and pump the source material to the building site through a pipeline.

The correct determination of the bulking factor is important for the design of a storage basin for a given quantity of (polluted) mud. If the bulking factor is too small, the volume of the storage basin may not be sufficient. Some values of the bulking factor \( f_{\text{bulk}} \) are given in Table 2.3 and 2.4.

Using a hopper dredger, the wet density after dredging can be easily determined from the hopper volume \( V_{\text{hopper}} \) and the water displacement \( V_{\text{water}} \) of the dredger: \( \rho_{\text{wet}} = (\rho_{\text{seawater}} V_{\text{water}}) / V_{\text{hopper}} \). The bulking factor can be determined if the in-situ wet bulk density of the source material is known from in-situ soil samples. Dredged mud from a harbour basin or from a navigation channel generally is very soft material. Practical experience at a hopper dredger working in the Holwerd channel in May 2016 shows that a long stick of 4 m can be easily driven through the mud (with wet density of 1350 kg/m\(^3\); Table 2.4) down to the hopper bottom (hull). Segregation of fine sand (dredged materials contain 25% fine sand in Holwerd channel) does not occur in the hopper as no sand layer was present at the hopper bottom.

### Table 2.3 Bulking factor of soils

<table>
<thead>
<tr>
<th>Soil type</th>
<th>In-situ (source)</th>
<th>Mechanical excavation</th>
<th>Hydraulic excavation/dredging</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Wet bulk density (kg/m(^3))</td>
<td>Wet bulk density at site (kg/m(^3))</td>
<td>Bulking factor (−)</td>
</tr>
<tr>
<td>Peat</td>
<td>970</td>
<td>980</td>
<td>1.5</td>
</tr>
<tr>
<td>Soft mud</td>
<td>1250</td>
<td>1200</td>
<td>1.25</td>
</tr>
<tr>
<td>Soft clay</td>
<td>1500</td>
<td>1400</td>
<td>1.25</td>
</tr>
<tr>
<td>Firm clay</td>
<td>1800</td>
<td>1700</td>
<td>1.1</td>
</tr>
<tr>
<td>Sand</td>
<td>2000</td>
<td>1900</td>
<td>1.1</td>
</tr>
</tbody>
</table>

### Table 2.4 Wet and dry bulk density of source and dredged materials (hopper dredging with overflow)

<table>
<thead>
<tr>
<th>Site</th>
<th>Fraction lutum/clay &lt; 4 μm</th>
<th>Fraction silt 4-63 μm</th>
<th>Fraction fine sand &gt; 63 μm</th>
<th>Wet and dry density of in-situ source material (kg/m(^3))</th>
<th>Wet and dry density of dredged mud in hopper (kg/m(^3))</th>
<th>Bulking factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>Holwerd channel Wadden Sea</td>
<td>10%</td>
<td>65%</td>
<td>25%</td>
<td>1450-1500; 700-750</td>
<td>1300-1350; 450-550</td>
<td>1.5</td>
</tr>
<tr>
<td>Noordpolderzijl channel Wadden Sea</td>
<td>35%</td>
<td>30%</td>
<td>35%</td>
<td>1400-1450; 600-700</td>
<td>not available</td>
<td>n.a.</td>
</tr>
</tbody>
</table>
2.5 Dredging and dumping costs

Dredging and dumping works are large-scale operations that generally continue for 24 hours per day and 7 days per week. The operators and other personnel are accommodated on board in shifts of 8 hours. Downtime for repairs and bad weather is generally less than 10%.

Overall dredging costs consist of:
- mobilisation/demobilisation
  - preparation/transportation of equipment;
  - preparation of building site and mooring places;
- equipment/machinery (including fuel);
- personnel;
- overhead (buildings; office people; factor 1.1 to 1.3).

Table 2.5 presents day-prices for various types of equipment.

The total price of a major dredging work is given by:

\[ P = f_{\text{overhead}} \left( f_{\text{bulking}} (V \ p) + D \right) \]

with:
- \( f_{\text{overhead}} = \) overhead factor (1.1 to 1.3);
- \( f_{\text{bulking}} = \) bulking factor (1 to 2);
- \( V = \) volume (m\(^3\)) of land fill to be made (in case of land reclamation project) or insitu (source) deposition volume in case of channel maintenance dredging;
- \( p = \) dredging/dumping unit price per m\(^3\);
- \( D = \) mobilisation/demobilisation costs.

In the case of dredging to make a land fill, the volume is given by the land fill site.
In the case of maintenance dredging of a navigation channel, the volume is given by the deposited material inside the channel.

For example: Hopper dredging project of mud for a land fill with \( V = 1.5 \) million m\(^3\) (fill volume of site); \( p = 5 \) Euro/m\(^3\); \( D = 0.5 \) million Euro; \( f_{\text{overhead}} = 1.3; f_{\text{bulking}} = 2 \) (mud), yields: \( P = 1.3 \left[ 2(1.5 \times 5) + 0.5 \right] \approx 20 \) million Euro

<table>
<thead>
<tr>
<th>Type of equipment</th>
<th>Price per day of 24 hours; including fuel (Euro per day)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Large hopper dredger (volume 10000 m(^3))</td>
<td>15000-20000</td>
</tr>
<tr>
<td>Small hopper dredger (volume 2000 m(^3))</td>
<td>5000-10000</td>
</tr>
<tr>
<td>Cutter dredger (5000 m(^3)/hour)</td>
<td>5000-10000</td>
</tr>
<tr>
<td>Backhoe/Crane-Grab dredger on jackup/spudpole ponton (300-700 m(^3)/hr)</td>
<td>5000-10000</td>
</tr>
<tr>
<td>Backhoe/Crane on land</td>
<td>1500-2500</td>
</tr>
<tr>
<td>Bulldozer</td>
<td>1500-2000</td>
</tr>
<tr>
<td>Large barge (600 m(^3))</td>
<td>700</td>
</tr>
<tr>
<td>Small barge (splitthull/bottom doors; 150 m(^3))</td>
<td>500</td>
</tr>
<tr>
<td>Multicat (pushboat)</td>
<td>1000-2000</td>
</tr>
<tr>
<td>Large dredge pump</td>
<td>500</td>
</tr>
<tr>
<td>Floating pipeline (1 km)</td>
<td>500</td>
</tr>
<tr>
<td>Labour/personnel</td>
<td>1000-1500</td>
</tr>
</tbody>
</table>

Table 2.5 Day-prices for dredging equipment and personnel
Figure 2.15  Land reclamation project of mud

Dredging and dumping for land reclamation project

**Given:** Land reclamation project of mud dredged from navigation channel (at about 1 km from the site).

Land reclamation is bounded by an enclosure dike of sand covered with a rock layer (building costs estimated to be about 10 million Euro), see Figure 2.15.

Site preparation costs (roads, drainage system, etc) are estimated to be about 5 million Euro.

Total volume of land reclamation site (area of 15 ha; total layer thickness of 10 m) = 1.5 million m³

Mobilisation costs = 0.5 million Euro; overhead factor \( f_{\text{overhead}} = 1.2 \)

Three solutions (based on conservative estimates) are considered:

1. mechanical dredging by grab, transportation by barges and dumping by barges (Table 2.6);
   - unit price of 8.5 to 10 Euro/m³;
   - bulking factor = 1.2 (about 1.2x1.5= 1.8 million m³ of mud+water is used to fill site);
   - overall price \( P = 1.2 \times \left(1.5 \times 10\right) + 0.5 \approx 22 \) million Euro;

2. mechanical dredging by grab, transport by barges, hydraulic dumping by auger-pump (Table 2.7);
   - unit price of 8.5 Euro/m³;
   - bulking factor = 1.4 (about 1.4x1.5= 2.1 million m³ of mud+ water is used to fill site);
   - overall price \( P = 1.2 \times \left(1.5 \times 8.5\right) + 0.5 \approx 22 \) million Euro;

3. hydraulic hopper dredging and hydraulic dumping/unloading (Table 2.8);
   - unit price of 5.5 Euro/m³;
   - bulking factor = 2 (about 2x1.5=3 million m³ of mud+water is used to fill the site);
   - overall price \( P = 1.2 \times \left(1.5 \times 4 + 5.5\right) + 0.5 \approx 15-20 \) million Euro.

Hydraulic methods are relatively cheap, but the bulking factors involved (Table 2.3) are relatively large resulting in a much larger volume that has to be dredged, transported and dumped.

The fully mechanical method 1 using large and small barges yields an overall price (about 22 million Euro) which is the same as that of the mechanical-hydraulic method 2.

The fully hydraulic method 3 is about 10% to 30% cheaper depending on the hopper size. Larger hoppers are more economic. The hopper size largely depends on locally available manoeuvring space. A negative aspect of hydraulic dredging is the large bulking factor involved because large quantities of water are required to pump the mud mixture. Thus: low-density mud is produced and additional drainage activities may be required to promote the consolidation process. Furthermore, the consolidation process to usable land will take much longer (factor 1.5 to 2) for the hydraulic method 3, which also has an economic price.

The overall costs of the land reclamation project including the construction of the enclosure dike and the site preparation (after sufficient consolidation) are: 10 + 22 (or 15) + 5 = 37 million Euro.

The land costs per m² is 30 to 37 \( 10^6 \times (15 \ 10^4\) ) = 200 to 250 Euro/m².

For reference: the construction costs of the new Rhine-Meuse coastal extension (effective area of 1000 ha for container storage; made of sand from offshore; completed in 2010) near the entrance to the Port of Rotterdam were about 3 \( 10^9 \) Euro (3 billion Euro) or 300 Euro/m².
## Table 2.6 Unit price of mechanical grab dredging, transport by large barges and dumping by small barges

<table>
<thead>
<tr>
<th>Type</th>
<th>Machinery and fuel costs (Euro per day)</th>
<th>Personnel costs</th>
<th>Costs (Euro per day)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 grab dredging pontoon</td>
<td>12000</td>
<td>8</td>
<td>8000</td>
</tr>
<tr>
<td>3 large barges (600 m³)</td>
<td>2000</td>
<td>6</td>
<td>6000</td>
</tr>
<tr>
<td>2 large multicats + 1 small multicat</td>
<td>5000</td>
<td>5</td>
<td>5000</td>
</tr>
<tr>
<td>1 backhoe on land</td>
<td>1000</td>
<td>2</td>
<td>2000</td>
</tr>
<tr>
<td>1 dredge pump + floating pipeline</td>
<td>1000</td>
<td>1</td>
<td>1000</td>
</tr>
<tr>
<td>Supporting equipment</td>
<td>2000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Local management/staff</td>
<td>4</td>
<td></td>
<td>6000</td>
</tr>
<tr>
<td><strong>Total costs</strong></td>
<td><strong>23000</strong></td>
<td><strong>26</strong></td>
<td><strong>28000</strong></td>
</tr>
<tr>
<td>Bare costs of dredging and dumping per m³</td>
<td>51000/6000</td>
<td></td>
<td>8.5 Euro per m³</td>
</tr>
<tr>
<td>Dredging costs including overhead (factor 1.1-1.3) per m³</td>
<td>9-11 Euro per day</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Machinery used: 1 Backhoe/ Crane ponton; 3 barges; 2 multicats; 1 backhoe on land; 1 small multicat; 1 dredge pump

Production= 300 m³/hour and 6000 m³/day (20 effective hours)

## Table 2.7 Unit price of mechanical dredging, transport by large barges and dumping by pump+pipeline

<table>
<thead>
<tr>
<th>Type</th>
<th>Machinery and fuel costs (Euro per day)</th>
<th>Personnel costs</th>
<th>Costs (Euro per day)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 hopper dredger (2000 to 3000 m³; length of 80 to 90 m)</td>
<td>10000-15000</td>
<td>8</td>
<td>8000</td>
</tr>
<tr>
<td>1 pumping station + floating pipeline (1 km)</td>
<td>2000</td>
<td>3</td>
<td>3000</td>
</tr>
<tr>
<td>1 bulldozer and 1 shovel</td>
<td>3000</td>
<td>4</td>
<td>4000</td>
</tr>
<tr>
<td>2 trucks</td>
<td>2000</td>
<td>4</td>
<td>4000</td>
</tr>
<tr>
<td>1 multicat/1 survey boat</td>
<td>3000</td>
<td>4</td>
<td>4000</td>
</tr>
<tr>
<td>Additional site-specific machinery inside compartment for stimulation of drainage (during final stage)</td>
<td>3000</td>
<td></td>
<td>3000</td>
</tr>
<tr>
<td>Supporting equipment</td>
<td>2000</td>
<td>1</td>
<td>1000</td>
</tr>
<tr>
<td>Local management/staff</td>
<td>3</td>
<td></td>
<td>3000</td>
</tr>
<tr>
<td><strong>Total costs</strong></td>
<td><strong>25000-30000</strong></td>
<td><strong>30</strong></td>
<td><strong>30000</strong></td>
</tr>
<tr>
<td>Bare costs of dredging and dumping per m³</td>
<td>60000/15000; 55000/10000 ≈ 4-5.5 Euro per m³</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Dredging costs including overhead (factor 1.1-1.3) per m³</td>
<td>4.5-7 Euro per day</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Machinery used: 1 hopper dredger (2000 m³); 1 bulldozer; 1 shovel; 2 trucks; 1 pipeline; 1 multicat; 1 survey boat.

Production= 10000 to 15000 m³/day based on 5 sailing trips per day (including downtime)

## Table 2.8 Unit price of hydraulic hopper dredging, transport by hopper and dumping through pipeline
3. Filling of compartments

The filling of the compartments at the land reclamation site can be executed mechanically (excavator; barge with bottom doors) or hydraulically (pumping). The selection of the most appropriate filling method depends primarily on the land use. To deliver short-term industrial land, it is essential to use high-density mud with a short consolidation period (mechanical dredging and dumping). For long-term natural land, low-density mud with a long consolidation period produced by hydraulic pumping methods is generally sufficient.

Mechanical filling yields the highest initial density of the soil, but the production rate is relatively low and the filling process is time consuming.

Hydraulic filling through a pipeline is the most efficient method. The mud slurry can be pumped into the compartment as close as possible to the bottom using a barge with vertically adjustable diffuser (Figures 3.1, 3.2) so that a thin high-density flow with low speed and turbulence is generated to avoid segregation. The mud flow will spread out horizontally in the form of a slurry tongue. There should be enough space (minimum 100 m) to allow this process to proceed in a slow and gradual way. The sediments that are carried in the sludge tongue, will mix with the lower part of the water column and then settle. The sludge should not be pumped at a high level in the water column because this will lead to strong segregation, dispersion and the settling of individual particles.

A spray barge (Figure 3.2) can also be used to cover the mud interface by a sand layer to speed up the consolidation process.
The filling process of a compartment for ecological purposes (nature) is schematically shown in Figure 3.3 and consists of:

- a) pumping of mud near the bottom;
- b) water is removed near the surface;
- c) new mud is pumped into compartment and mud interface gradually settles;
- d) more mud is pumped into compartment and mud interface gradually settles;
- e) mud settles into higher density layer;
- f) mud settles into higher density layer;
- g) mud crust is formed at the surface.

![Figure 3.3 Schematic filling of compartment](image)

The consolidation of the top soil layer depends on the construction method:

- **mechanically placed**: the soft soil will consolidate in about 3 to 5 years to moderately firm soil with a wet density of 1500 to 1700 kg/m³;
- **hydraulically pumped**: the top layer will dry out and from cracks, depending on the weather conditions; the crust formation cycle is about 5 to 10 years: from fluid mud suspension under a film of water (wet bulk density of 1200 kg/m³) to soft soil under water (1400 kg/m³) in about 5 years and to moderately firm soil above water (1500 to 1700 kg/m³) in about 3 to 5 years.

Eventually, there will be a crust of moderately firm soil (1700 kg/m³) with a thickness of 0.5 m on top of a package of muddy soil (clay/peat/sand) with a wet density of 1400 kg/m³.
4. Crust formation, cover layer and drainage systems

4.1 General

The required bearing capacity of the soil surface is approximately 0.5 to 1 kg/cm² (5 to 10 tons/m²) to allow small animals and people to walk over the soil surface. The required bearing capacity for machinery (backhoe) is up to 3 kg/m² (30 ton/m²). Generally, special soil stabilisation works are required to improve the top layer of soft soils in order that heavy machinery can be used for building activities.

Table 4.1 gives an overview of various types of loads and bearing capacities of soils. The crust of the soil (top layer of 0.5 m) must have a wet density of about 1500 to 1600 kg/m³ in order to have a bearing capacity of 0.5 to 1 kg/cm². The natural crust formation process is highly dependent on the density of the top soil layer and weather/climatological conditions (sun drying).

Two situations are possible:
1. soft mud soil with initial density of 1300 to 1400 kg/m³ (mechanically placed using an excavator);
2. soft mud slurry with initial density of 1100 to 1200 kg/m³ (hydraulically pumped using a pipeline).

<table>
<thead>
<tr>
<th>Type</th>
<th>Load (kg)</th>
<th>Loading area (cm²)</th>
<th>Load per unit area (kg/cm²)</th>
<th>Bearing capacity (kg/cm²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Small animals</td>
<td>5</td>
<td>10 to 20</td>
<td>0.25 to 0.5</td>
<td></td>
</tr>
<tr>
<td>Male person</td>
<td>80</td>
<td>40 to 80</td>
<td>0.5 to 1</td>
<td></td>
</tr>
<tr>
<td>Light machinery on wide tires (1 ton per tire)</td>
<td>1000</td>
<td>30x10</td>
<td>1 to 3</td>
<td></td>
</tr>
<tr>
<td>Machinery on flat plates (10 to 30 ton/m²)</td>
<td>30000</td>
<td>100x100</td>
<td>1 to 3</td>
<td></td>
</tr>
<tr>
<td>1. Very soft muddy-watery mixture; wet density = 1300-1400 kg/m³; dry density= 450-600 kg/m³</td>
<td></td>
<td></td>
<td></td>
<td>0.1-0.3</td>
</tr>
<tr>
<td>2. Soft muddy (buttery-type) soil; wet density = 1400-1500 kg/m³; dry density= 600-750 kg/m³; (from 1 after 6 months of consolidation; layer of 0.5 m); not walkable for people</td>
<td></td>
<td></td>
<td></td>
<td>0.3-0.5</td>
</tr>
<tr>
<td>3. Soft malleable soil; wet density = 1500-1600 kg/m³; dry density= 750-900 kg/m³; walkable for people with special boots</td>
<td></td>
<td></td>
<td></td>
<td>0.5-1</td>
</tr>
<tr>
<td>4. Firm mudtype soil; wet density = 1700-1900 kg/m³; dry density= 900-1100 kg/m³; walkable for people</td>
<td></td>
<td></td>
<td></td>
<td>1.5</td>
</tr>
<tr>
<td>5. Sand; wet density = 1900-2000 kg/m³</td>
<td></td>
<td></td>
<td></td>
<td>&gt; 10</td>
</tr>
</tbody>
</table>

Table 4.1 Load and bearing capacity of soil (Van der Veen, 1962)

4.2 Top layer consisting of soft soil (1300 to 1400 kg/m³) by mechanical placement

A top soil layer with a relatively high initial wet density of 1300 to 1400 kg/m³ can only be obtained by mechanical placement using a crane or an excavator (Haasnoot and De Vos 2010; Roukema et al. 1998). The soil with a maximum layer thickness of about 2 m should be placed mechanically with a surface level of about 1 m above the surrounding water surface or ground water table. The soil surface will dry out by evaporation and by plants/vegetation extracting moisture from the soil, see Figure 4.1. This reduces the water content and the mineral and organic particles are attracted to each other by capillary forces resulting in a more firm soil. The drying process will result in (visual) cracking if the percentages of clay and organic materials are sufficiently high. Cracking stimulates the penetration of air, which enhances chemical and biological aging (rippling) processes depending on the soil composition and environmental conditions.

Aging (rippling) is a natural and irreversible process of drying and oxidation, through which the largely anaerobic mud sludge turns into a more compact and permeable soil material. Thus, the soil material changes gradually from a thin, wet slurry into a more solid and firm soil. Depending on the initial dry mass and the physical composition, the soil volume may reduce with 30% to 70%.
Figure 4.1  Crust formation of soft muddy soil using mats

The ripening process can broadly be divided into three sub-processes:
- physical aging (decrease of water content with cracking and subsidence of the soil);
- bio/chemical aging with oxidative conversions (oxidation of organic matter) and
- (micro) biological aging with microbial degradation.

Aging can be significantly promoted by rapid drainage of surface water (due to precipitation), so that it cannot penetrate into the soil. This requires efficient drainage of surface water by a system of gullies and ditches or through drainage pipes in the bottom.

The most important processes in the physical ripening processes are: evaporation, precipitation and seepage/drainage. Generally, the drainage of water through seepage will be very low (less than 0.1 mm/day), because the water level in the compartments will be equal or only slightly higher than that of the surrounding water.

The evaporation of surface water in sunny summer periods with temperatures between 20° and 25° is of the order of 5 mm/day. If the annual rainfall is of the order of 1000 mm or 3 mm/day, the dominant evaporation in the summer can lead to a relatively dry top soil layer.

The evaporation is determined by two factors:
- open-ground evaporation (evaporation) and
- crop/plant evaporation (transpiration).

Evaporation is limited to the top layer of the soil and will significantly decrease in time due to crust formation. The drying of the deeper layers strongly depends on the drainage system.

In a dry summer period of about 4 weeks, a layer of water of about 100 to 150 mm can be removed through evaporation, resulting in a solid crust of about 1500 kg/m³. One relatively long dry period is extremely important. The soil is then sufficiently firm for the planting of vegetation. Aging can be accelerated by sowing and growing of reed. Reed can grow in soft soil and stimulate the aging process by transpiration processes. Cracks will occur after a long dry period, see Figure 4.3.

The stages of soil improvement by reed growth are (see Figure 4.2):
- reed is sowed/planted on the thin crust after an initial dry period;
- reed accelerates dewatering under the dry crust resulting in quicker consolidation; crust becomes thicker and sinks;
- new mud sludge can be pumped on the sinking crust.
In this way the soft holocene surface layer can consolidate in about 2 to 3 years to more firm soil with a wet density of about 1500 kg/m³. The total layer thickness will decrease by 40% to 50%.

Figure 4.2  *Crust formation through vegetation*

Figure 4.3  *Mud cracks in crust surface*
4.3 Top soil layer consisting of soft mud slurry (1200 to 1300 kg/m$^3$) by hydraulic method

If the mechanical placement of the top soil layer is not feasible, the only other option is the hydraulic placement of mud through a pipeline. The hydraulic pumping of a mud slurry will result in a relatively low density of the soil of about 1200 kg/m$^3$. The water/mud level in the compartments (closed ring dike) should be at least 1 m above that of the surrounding waters. The drying process depends on the relative magnitudes of the rainfall, evaporation and drainage of water through seepage flow.

The amount of seepage water depends on the water level difference between the inside and the outside of the compartments and the permeability of the soil plus (sandy) ring dikes. The seepage flow out of a compartment of 50 ha can be calculated with: $Q = k \times h \times (H/B) \times L$ where $k = \text{permeability coefficient} < 10^{-6} \text{ m/s}$ for muddy soils, $h = \text{layer thickness of soil} = 5 \text{ m}$, $H = \text{water level difference} = 1 \text{ m}$, $B = \text{dike width} = 10 \text{ m}$, $L = \text{dike length} = 5000 \text{ m}$.

This gives a seepage flow: $Q = 0.0025 \text{ m}^3/\text{s} = 200 \text{ m}^3/\text{day}$ or a water level reduction within the compartment of about 0.4 mm/day for a surface of 50 ha. As the rainfall and evaporation are of the order of 2 to 5 mm/day, the contribution of seepage flow is negligibly small.

Only in a very dry summer period of approximately 4 weeks, there will be adequate evaporation to remove a layer of water of 100 to 150 mm through evaporation. When the soil becomes dry after a sunny summer period, the top layer consists of loose mineral and organic particles surrounded by bound water films. The top layer has a soft consistency with a density of 1400 to 1500 kg/m$^3$. Possibly there will be a thin and moderately firm crust. The soil will have insufficient capacity for the planting of vegetation.

If the amount of water extracted from the top layer by evaporation is compensated by rainfall, the moisture content of the top layer will not go down and hence aging can not occur. In this situation, the soil surface will sink by internal consolidation with an in thickness growing water layer on top of it. The surface water layer will continuously have to be removed through drainage (pumping), so that the crust formation can proceed in the next dry period. In a summer with intensive rainfall, there will be a continuous thin layer of water on the soil surface. Regular refilling with mud slurry will be necessary.

The crust formation cycle in this rainy regime will be about 5 to 10 years:
- 5 years from soft fluid mud under water (wet density 1200 kg/m$^3$) to soft soil under water (1400 kg/m$^3$);
- 3 to 5 years from soft soil under water to moderately firm soil above water (1500 kg/m$^3$).

Eventually, there will be a crust of moderately firm soil (1500 kg/m$^3$) with a thickness of approximately 0.5 to 1 m resting on package of soft subsoils (clay/peat/sand) with a wet density of 1400 kg/m$^3$. The crust will not consolidate further due to regular addition of rainwater. Only in long dry summer periods the density of the top soil layer can increase further. The soft subsoil will consolidate in about 100 years to firm soil with a density of 1700 kg/m$^3$.

4.4 Stabile soil cover layer (cap)

If the land reclamation of soft mud is on the long term to be used for industrial activities, it is necessary to make a stable soil cover layer on top of the soft soil package. Once, a stable cover layer is present, machinery can be used for drainage activities (vertical drains or sand columns).

Two methods can be used:
- sand layer (at least 2 m) supplied by trucks over land or by hydraulic pumping (through pipeline) using sand supplied by hopper dredging from offshore;
- soil stabilisation using cement mixing ($www.allu.net$).
Soil stabilisation is a geotechnical method (Figure 4.4) to improve the bearing capacity and stability of soft muddy soils. Clay, peat, silt, sediment, sludge, or dredged material can be transformed into solid ground. The base material should have a wet bulk density of at least 1500-1600 kg/m³. The method relies on thoroughly mixing of the soft top layer (over 2 to 8 m) with cement-type binders into subject material while the material remains in-place or insitu. After drying/binding, a geotextile is laid and a layer of sand and/or gravel (1 m) is placed by trucks, on which the machinery can move forwards to extend the stable soil area. Production rates are up to 1000 m² per day (prices are about 15-30 Euro/m²).

![Soil stabilisation method](www.allu.net, Finland)

### 4.5 Drainage systems

#### 4.5.1 Natural drainage systems

The natural consolidation of soft soil proceeds most efficient if the soil surface is made as high as possible (order of 1 m) above the surrounding water level to create a water level difference promoting seepage/drainage flows. This requires the presence of permeable (sand) ring dikes, narrow drainage channels, adjustable gates and open connections between the drainage channels, see Figure 4.5. Small-scale wind mills can also be used for the removal of surface water.

![Drainage of water](image)
A small windmill has a drainage capacity during normal wind conditions (Beaufort scale 3 to 5) of about 10 l/s or 400 m³/day assuming 10 working hours. This gives a lowering of the water surface level with about 1 mm/day for a compartment area of 50 ha.

An adjustable gate can be used to maintain a certain preset water level in a compartment. A gate having a width of 1 m has a maximum drainage capacity of about 50 l/s or 4000 m³/day for a water level difference of about 0.5 m, which is a lowering of the water surface of about 5 mm/day for a compartment of 50 ha.

### 4.5.2 Artificial drainage systems

Landuse of soft soils without artificial soil drainage is usually impractical due to unpredictable long-term settlement. Simple surcharging as a soil consolidation method can take many years. Soil consolidation using prefabricated vertical drains (also known as band drains, PV drains or wicks drains; www.wicks.nl) can rapidly increase settlement rates and cut project durations drastically (factor 5 to 10), see Figure 4.6.

**Figure 4.6** Vertical drainage system

The prefabricated vertical drain core is made of high quality flexible polypropylene which exhibits a large water flow capacity in the longitudinal direction of the core via preformed grooves or water channels on both sides of the core. Each vertical drain can provide a greater vertical discharge capacity than a sand column with a diameter of 0.15 m. The prefabricated vertical drain core is tightly wrapped in a geotextile filter jacket of spun-bonded polypropylene which has a very high water permeability while retaining the finest of soil particles. Both the core and geotextile filter jacket have high mechanical strength, a high degree of durability in most environments, and high resistance to chemicals, and micro-organisms. Prefabricated vertical drains are installed vertically to depths of 50 m. The water (due to overpressure) flows through the vertical drain out of the soil and evaporates freely. This flow may be either up or down to intersecting natural sand layers or to the surface consisting of a horizontal sand drainage layer. The water in the soil has only to travel the distance to the nearest prefabricated vertical drain to reach a free drainage path. The drains are placed very close together at spacings of 1 to 2 m (covered area of 1 strip is 1 to 4 m²). The price of strip material plus placement is about Euro 0.8-1 per m length of strip.

In situations with a stable sand cover layer, the artificial drainage strips can be made by cranes/rigs operating over land. In situations with a soft consolidating soil, it may not be feasible to work over land. In stead of that, it may be feasible to pump water into the compartment to create a water depth of about 1.5 m in which a barge can operate. The vertical drain strips can be installed under water from the barge, see Figure 4.7.
Required equipment: 1 crane/rig, 1 truck and 3 workers
Production rate of strips at spacing of 2 m (4 m²), length=15 m: 700 strips per day or 2000-3000 m² per day
Unit price (incl. mobilisation): Euro 10000 per day or Euro 3-5 per m²

Figure 4.7  *Vertical drainage strip installed under water from a barge*
5. Consolidation of soft muddy soil to firm soil

5.1 Consolidation process

The sedimentation and consolidation process of suspended mud at the bottom of the water column can be divided into four phases (see also Figure 5.1):

- hindered settling in which the particles and flocs move slowly downward hindered by the return flow of water displaced by the moving sediments; the sediment concentrations are approximately 10 to 200 kg/m³ (wet density of 1010 to 1150 kg/m³); settlement is of the order of 50% of the initial layer thickness; time scale of days;
- initial consolidation phase, which is a transitional phase (with thick fluid mud) in which the particles and flocs make contact with each other resulting in a matrix network structure (gelling concentration) and significant decrease of the effective settling velocity (wet density 1150 to 1400 kg/m³); settlement is of the order of 20% of the initial layer thickness; time scale is weeks to 1 year;
- primary consolidation phase in which there is a slow building up of contact forces (grain stresses) and pore water is driven out; an initial soil structure (matrix) is formed with small dewatering channels (cracks) through which water can escape; the dry bulk density at the onset of the primary consolidation phase is about 300 to 400 kg/m³; the dry density at the end of the primary phase is about 600 to 700 kg/m³ (wet bulk density varies in the range of 1400 to 1500 kg/m³; moderately firm soil), depending on the type of mud and percentage of sand; time scale of weeks to months;
- secondary consolidation stage in which the soil network structure is strengthening (development of firm soil) with wet bulk density values in the range of 1500 to 1600 kg/m³; settlement is smaller than 5% of the initial layer thickness; the time scale depends on the upper load and drainage rate. If no drainage is present, the typical time scale is of the order of 1 to 100 years.

Consolidation is a process that requires space, time and drainage. Space because the fluid mud has to be pumped into the compartments as gradual as possible with low velocities to suppress turbulence. Time because the extrusion of pore water goes slow. Experimental results show that the consolidation (soil surface sinking) of hydraulically filled compartments is of the order of 0.01-0.03 m/month (0.3 m/year) without drainage during the initial stages.

Table 5.1 provides an overview of the volume densities and consolidation times of different types of soil.
### Table 5.1: Overview of soil types, volume densities and consolidation times

<table>
<thead>
<tr>
<th>Soil Type</th>
<th>Dry Volume Weight $c_{dry}$ (kg/m$^3$)</th>
<th>Wet Volume Weight $\rho_{wet}$ (kg/m$^3$)</th>
<th>Specific Volume Weight $\rho_s$ (kg/m$^3$)</th>
<th>Porosity $\eta$ (%)</th>
<th>Consolidation time up to 95% of the final value $T_{end}$ (years)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Soft to firm soil (under water)</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sand (0.062 to 2 mm)</td>
<td>1550± 50</td>
<td>1950± 50</td>
<td>2650</td>
<td>42%</td>
<td>$$&lt;&lt; 1 \text{ year}$$ (very solid soil)</td>
</tr>
<tr>
<td>Silt (0.032 to 0.062 mm)</td>
<td>1350 ± 50</td>
<td>1850± 50</td>
<td>2650</td>
<td>50%</td>
<td>$&gt; 100 \text{ years to } \rho_{wet} = 1900 \text{ (firm soil)}$</td>
</tr>
<tr>
<td>Moderate firm Holocene clay in soil under (fresh) water; mechanically placed</td>
<td>800±100</td>
<td>1500±50</td>
<td>2650</td>
<td>70%</td>
<td>$&gt; 100 \text{ years to } \rho_{wet} = 1700 \text{ kg/m}^3$ (moderately firm soil)</td>
</tr>
<tr>
<td>Soft Holocene clay in soil under (fresh) water; mechanically placed</td>
<td>600±100</td>
<td>1350±50</td>
<td>2300</td>
<td>75%</td>
<td>$&gt; 100 \text{ years to } \rho_{wet} = 1500 \text{ (very moderately firm soil)}$</td>
</tr>
<tr>
<td><strong>Fluid mud mixtures (under water)</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pure clay (kaolinite) in salt water, layer = 0.3 m</td>
<td>100</td>
<td>1100</td>
<td>2650</td>
<td>95%</td>
<td>1 week to $\rho_{wet} = 1400$ (soft soil)</td>
</tr>
<tr>
<td>Bangkok mud in salt water, layer= 1 m</td>
<td>200</td>
<td>1150</td>
<td>2500</td>
<td>92%</td>
<td>0.75 years to $\rho_{wet} = 1300$ (soft soil)</td>
</tr>
<tr>
<td>Bangkok mud in salt water, layer= 2 m</td>
<td>200</td>
<td>1150</td>
<td>2500</td>
<td>92%</td>
<td>3 years to $\rho_{wet} = 1300$ (soft soil)</td>
</tr>
<tr>
<td>Slufter mud in salt water, bottom layer of 1 m with upper layers of 15 to 20 m</td>
<td>375</td>
<td>1250</td>
<td>2500</td>
<td>85%</td>
<td>10 years to $\rho_{wet} = 1400$ (soft soil)</td>
</tr>
<tr>
<td>Sandy mud mixture (fine sand/clay/20% peat) after hydraulic dredging</td>
<td>500±100</td>
<td>1300±50</td>
<td>2300</td>
<td>80%</td>
<td>1 year to $\rho_{wet} = 1800$ (moderately firm soil)</td>
</tr>
<tr>
<td>Mud (clay/20% peat) mixture after hydraulic dredging</td>
<td>300±100</td>
<td>1200±50</td>
<td>2300</td>
<td>85%</td>
<td>5 years to $\rho_{wet} = 1400$ (soft soil)</td>
</tr>
<tr>
<td>Mud (clay/50% peat) mixture after hydraulic dredging</td>
<td>200±100</td>
<td>110±50</td>
<td>1800</td>
<td>90%</td>
<td>$&gt; 10 \text{ years to } \rho_{wet} = 1200$ (soft soil)</td>
</tr>
<tr>
<td><strong>Soft soil (above water)</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Soft Holocene clay/20% peat above ground water; mechanically placed</td>
<td>600±100</td>
<td>1350±50</td>
<td>2300</td>
<td>75%</td>
<td>3 years to $\rho_{wet} = 1500$ (moderately firm soil above water)</td>
</tr>
<tr>
<td>Soft Holocene peat/20% clay above ground water; mechanically placed</td>
<td>300±100</td>
<td>1050±50</td>
<td>1300</td>
<td>75%</td>
<td>3 years to $\rho_{wet} = 1200$ (moderately firm soil above water; peat like)</td>
</tr>
</tbody>
</table>

Peat = mixture of organic/planttype materials with specific mass of 1000 ±100 kg/m$^3$; dry on land after 0.5 to 1 year.
Formulas: $\rho_{wet} = \rho_{water} + [(\rho_s - \rho_{water})/\rho_s] c_{dry} = 1000 + 0.62 c_{dry}$; $\eta = 1 - c_{dry}/\rho_s$
Figure 5.2 shows the consolidation of the wet and dry density of mud in salt water for various types of mud (Van Rijn 1993, 2006). To obtain a wet density above 1300 kg/m³, a consolidation time of at least 100 days (3 months) is required. The results of Bangkok-mud (with a very large fraction of clay/lutum) show that a mud layer of 1 m consolidates significantly faster than a layer of 2 m. The lower layers of Slufter-mud situated under a total soil package of 15 to 20 m (depot Rotterdam 1987-1994; Wichman 1995) have consolidated in about 7 years to a wet density of 1400 kg/m³. The density of the top layer does not increase much in time (1250 kg/m³) due to regular filling with new mud slurry. Laboratory test results show that the thickness of the mud layers should be limited (not more than 2 to 4 m thick) as the dewatering process proceeds slower for increasing thickness (without additional drainage).

![Figure 5.2: Wet and dry bulk density of mud as a function of time; various types of muds](image)

Figure 5.3 shows settling test results (column height of 1 m) of fine sediments from various locations in the Rio de la Plata, Uruguay (Fossati et al., 2007). The initial conditions and sediment properties are shown in Table 5.2. Based on these results, it can be concluded:

- very silty mud (R<0.2): duration of the hindered settling process is about 1 hour and the relative mud height is about \( h_{\text{mud}}/h_0 = 0.15 \) to 0.2 at the end of the hindered settling process; after 60 days of consolidation: \( h_{\text{mud}}/h_0 = 0.13 \);
- very clayey mud (R>0.45): duration of the hindered settling process is about 2 to 4 hours and the relative mud height is about \( h_{\text{mud}}/h_0 = 0.45 \) to 0.65 at the end of the hindered settling process; after 60 days of consolidation: \( h_{\text{mud}}/h_0 = 0.25-0.30 \).

Thus: the consolidation process proceeds much slower for muds with a large fraction of clay/lutum (R>0.5).

**Table 5.2** Settling test data, Rio de la Plata, (Fossati et al., 2007)

<table>
<thead>
<tr>
<th>Test</th>
<th>Initial concentration ( c_0 ) (kg/m³)</th>
<th>Initial column height ( h_0 ) (m)</th>
<th>Test duration (days)</th>
<th>Mud composition</th>
<th>Salinity (kg/m³)</th>
<th>Duraion hinder settling (hours)</th>
<th>Relative mud height at end of hindered settling (%)</th>
<th>Relative mud height after 60 days (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>E1T1</td>
<td>98</td>
<td>1.01</td>
<td>23</td>
<td>12% sand, 45% silt, 31% clay</td>
<td>0.45</td>
<td>3.5</td>
<td>0.45</td>
<td>0.23</td>
</tr>
<tr>
<td>E2T1</td>
<td>100</td>
<td>1.003</td>
<td>162</td>
<td>19% sand, 71% silt, 10% clay</td>
<td>0.13</td>
<td>&lt;1</td>
<td>0.15</td>
<td>0.13</td>
</tr>
<tr>
<td>E2T2</td>
<td>100</td>
<td>0.973</td>
<td>162</td>
<td>0% sand, 66% silt, 33% clay</td>
<td>0.50</td>
<td>4</td>
<td>0.52</td>
<td>0.28</td>
</tr>
<tr>
<td>E2T3</td>
<td>100</td>
<td>1.032</td>
<td>162</td>
<td>18% sand, 65% silt, 14% clay</td>
<td>0.16</td>
<td>&lt;1</td>
<td>0.20</td>
<td>0.14</td>
</tr>
<tr>
<td>E3T1</td>
<td>100</td>
<td>0.957</td>
<td>65</td>
<td>56% sand, 44% silt, 47% clay</td>
<td>0.77</td>
<td>2</td>
<td>0.60</td>
<td>0.27</td>
</tr>
<tr>
<td>E3T2</td>
<td>100</td>
<td>0.962</td>
<td>53</td>
<td>56% sand, 44% silt, 47% clay</td>
<td>0.45</td>
<td>2</td>
<td>0.65</td>
<td>0.30</td>
</tr>
<tr>
<td>E3T3</td>
<td>100</td>
<td>0.962</td>
<td>51</td>
<td>68% sand, 31% silt, 47% clay</td>
<td>0.65</td>
<td>2.5</td>
<td>0.65</td>
<td>0.24</td>
</tr>
</tbody>
</table>

\( R = \% \text{clay}/\% \text{silt} \); Relative mud height = \( h_{\text{mud}}/h_0 \).
5.2 Effect of upper load (thin sand layer) on consolidation of mud

Dankers (2006) has carried out tests in a sedimentation column with small amounts of fine and coarse sand (approximately 10 grams; 0.11 mm; 0.36 mm) which are settling on a liquid mud layer (in salt water). The dry density of the top mud layer was 50 to 100 kg/m$^3$. The sand concentration was < 10 kg/m$^3$.

The test results show:

- falling sand grains disturb the mud surface; the sand grains penetrate through the mud surface and slowly move down through small dewatering channels (cracks);
- small accumulations (pockets) of sand in the mud layer; smaller sand particles are partly stopped by the mud structure; larger sand grains reach the bottom of the column;
- the effective settling velocity of the sand particles decreases by a factor of 3 to 10 depending on the mud concentration; the consolidation of the mud (sink velocity of mud surface) increases by 10%.

Based on these results, the presence of a thin sand layer (0.5 m) on top of a soft mud layer may accelerate the consolidation of the mud layer by improving the permeability so that the pore water can be driven out more quickly. Torfs et al. (1996) have also shown that sand layers can speed up the consolidation process.
5.3 Gibson consolidation model for soft mud soils

The most general model for self-weight consolidation of soft soils is the Gibson model, which can be derived from the vertical continuity equation of sediments with $\phi =$ volume concentration (function of $z$) and $w_s =$ settling velocity (function of $z$), as follows (Gibson et al., 1967; Winterwerp and Van Kesteren, 2004; Merckelbach, 1996):

$$\frac{\partial \phi}{\partial t} - \frac{\partial (w_s \phi)}{\partial z} = 0$$  \hspace{1cm} \text{(5.1)}

$$\frac{\partial \phi}{\partial t} \left( \frac{\partial e}{\partial t} + \frac{\partial (w_s/(1+e))}{\partial z} \right) = 0$$

$$-\frac{1}{(1+e)^2} \frac{\partial e}{\partial t} - \frac{\partial (w_s/(1+e))}{\partial z} = 0$$

$$\frac{\partial e}{\partial t} + (1+e)^2 \frac{\partial (w_s/(1+e))}{\partial z} = 0$$  \hspace{1cm} \text{(5.2)}

The relative pore velocity (Figure 5.4):

$$w_{pore,\text{effective}} = w_{pore} - w_s$$  \hspace{1cm} \text{(5.3)}

$w_{pore,\text{effective}} =$ pore velocity relative to downward moving sediment particle;

$w_{pore} =$ pore velocity due to water displacement of sediment particles.

**Figure 5.4 Definition sketch of vertical velocities**

Continuity: $\eta w_{pore} + (1-\eta)w_s = 0$; $w_{pore} = -((1-\eta)/\eta)w_s$; or $\eta w_{pore} + w_s - \eta w_s = 0$ or $\eta (w_{pore} - w_s) = -w_s$ (5.4)

Darcy law: $w_{\text{darcy}} = \eta w_{\text{pore, effective}} = \eta (w_{pore} - w_s) = -w_s = -(k/\rho_w g) \frac{\partial p}{\partial z}$ (5.5)

$w_{\text{darcy}} =$ vertical upward Darcy velocity is equal to the downward settling velocity; slowly settling flocs create an excess pore pressure which drives an upward pore flow.

Pore excess pressure:

$$\sigma = \rho + \sigma_s = \rho_\text{static} + \rho_e + \sigma_s = \rho_w(h-z) + \rho_e + \sigma_s$$

$$\frac{\partial \sigma}{\partial z} = -\rho_w g + \frac{\partial p}{\partial z} + \frac{\partial \sigma_s}{\partial z}$$

$$\frac{\partial p}{\partial z} = \rho_w g + \frac{\partial \sigma}{\partial z} - \frac{\partial \sigma_s}{\partial z}$$  \hspace{1cm} \text{(5.6)}

Soil stress:

$$\sigma = \rho_\text{soil} g z = \eta \rho_w + (1-\eta)\rho_e/\rho_w + (1/(1+e))\rho_s$$

$$\frac{\partial \sigma}{\partial z} = \frac{\partial \sigma_s}{\partial z} + \frac{\partial \rho_p}{\partial z} - \frac{\partial \sigma_f}{\partial z}$$  \hspace{1cm} \text{(5.7)}

The Darcy velocity is:

$$w_{\text{darcy}} = -k(\rho_w g) \left[ \frac{\partial p}{\partial z} - (k/\rho_w g) \left[ \rho_w g + \frac{\partial \sigma}{\partial z} - \frac{\partial \sigma_s}{\partial z} \right] \right]$$

$$w_{\text{darcy}} = -k(\rho_w g) \left[ \rho_w g - \frac{\partial \sigma}{\partial z} + \frac{\partial \rho_p}{\partial z} \right]$$

$$w_{\text{darcy}} = -k + \left[ k/(\rho_w g) \left[ \frac{\partial \sigma_s}{\partial z} - k/(\rho_w g) \left[ \frac{\partial \sigma_f}{\partial z} + \frac{\partial \rho_p}{\partial z} \right] \right] \right] \left[ \frac{\partial \rho_p}{\partial z} - (k/\rho_w g) \left[ \rho_w g + \frac{\partial \sigma}{\partial z} - \frac{\partial \sigma_s}{\partial z} \right] \right]$$  \hspace{1cm} \text{(5.8)}
The settling velocity can be replaced by:
\[ \frac{w_s}{w_{darcy}} = k - \frac{k (1+e)}{(1+e) \rho_w + (1/(1+e) \rho_s)} \]
\[ \frac{w_t}{w_{darcy}} = k + \frac{k (1+e)}{(1+e) \rho_w + (1/(1+e) \rho_s)} \]

Using Equations (5.3) to (5.9), Equation (5.2) can be described as:
\[ \frac{\partial e}{\partial t} + (1+e)^2 \left( \frac{\rho_s - \rho_w}{\rho_w} \right) \frac{\partial \left[ k/(1+e)^2 \right]}{\partial z} + \left[ (1+e)^2 / (\rho_w) \right] \frac{\partial \left[ (k/(1+e)) \right]}{\partial z} = 0 \] (5.10)

Equation (5.10) is known as the Gibson-equation. The variables \( k, e \) and \( \sigma_s \) are three unknown functions of \( z \) and \( t \). Numerical solution can be simplified by assuming that \( e \) and \( k \) are known power functions (with calibration/fit coefficients) of the grain stress (\( \sigma_s \)). Numerical solution is difficult because of the moving upper interface. Therefore, Equation (5.10) is rewritten in a moving reference system (see Winterwerp and Van Kesteren, 2004; Merckelbach, 1996).

with:
\[ \phi = \text{volume concentration} = c/\rho_s = 1/(1+e); \phi = 1-\eta; \]
\[ c = \text{mass concentration} = (\rho_d / \rho_s); \]
\[ e = \text{void ratio} = \eta/(1-\eta) = (\rho_s / c); \]
\[ \eta = \text{porosity factor} = e/(1+e) = 1-\phi \]
\[ w_s = \text{settling velocity of mud particles/flocs relative to fixed datum} \] (negative value in downward direction);
\[ w_{pore} = \text{pore water velocity relative to fixed datum}; \]
\[ w_t = \text{settling velocity relative to fixed datum}; \]
\[ \sigma = \text{soil stress}; \sigma_s = \text{grain stress}; p = \text{pore water pressure}; p_e = \text{excess pore water pressure}. \]

In the early phase of the consolidation process, the grain stresses is relatively small (\( \partial \sigma_s / \partial z \approx 0 \)) and can be neglected resulting in:
\[ \frac{\partial e}{\partial t} + (1+e)^2 \left( \frac{\rho_s - \rho_w}{\rho_w} \right) \frac{\partial \left[ k/(1+e)^2 \right]}{\partial z} = 0 \] (5.11)

Equation (5.11) is known as the Equation of Kynch.

In the end phase of the consolidation process, the vertical gradient of the deformations are small (\( \partial e / \partial z \approx 0 \)) and the permeability is almost constant resulting in:
\[ \frac{\partial e}{\partial t} + \left[ k(1+e) / (\rho_w) \right] \frac{\partial \sigma_s / \partial z}{\partial z} = 0 \] (5.12)

Using: \( \Delta e = \Delta \sigma / (1+e) = -m_e \Delta \sigma_s \) or \( 1/m_e = - (1+e) \partial \sigma_s / \partial e \)

The consolidation coefficient is: \( c_e = k/(\rho_w m_e) = -(1+e)k / (\rho_w) \partial \sigma_s / \partial e \)

Equation (5.12) can be described as:
\[ \frac{\partial e}{\partial t} - [c_e \partial e / \sigma_s] \partial \sigma_s / \partial z^2 = 0 \]
\[ \frac{\partial e}{\partial \sigma_s} = c_e \partial \sigma_s / \partial t - c_e \partial \sigma_s / \partial z = 0 \]
\[ \frac{\partial \sigma_s / \partial t - c_e \partial \sigma_s / \partial z = 0 \]
\[ \frac{\partial \sigma_s / \partial t - c_e \partial \sigma_s / \partial z = 0 \] (5.13)

with: \( \sigma_s = \sigma_s + \sigma_s = \text{grain stress}; \sigma_s = \text{grain stress before loading (constant); } \sigma_s = \text{excess grain stress}. \)

Equation (5.13) is known as the classical Terzaghi consolidation equation (in terms of the grain stress) for firm soils, see Equation (5.34).
Gibson et al. (1981) have linearized Equation (5.10) as follows:

\[
\frac{\partial e}{\partial t} - \mu \frac{\partial^2 e}{\partial z^2} - \lambda (\rho_s - \rho_w) g = 0
\]

(5.14)

\[
\mu = -\frac{k}{(\rho_w g)} \left[ \frac{1}{1+e} \right] \frac{\partial \sigma_s}{\partial e}
\]

\[
\lambda = -\frac{\partial (\frac{\partial e}{\partial \sigma_s})}{\partial e}
\]

k= function(e)

\[k=\text{function}(e)\]

5.4 Hindered settling and initial consolidation of soft mud soils

5.4.1 Hindered settling processes

In low-concentration flows, the most dominant process is flocculation (forming of flocs) resulting in an increase of the settling velocity, see Figure 5.5. In high-concentration flows the suspended sediment particles can not settle freely due to the presence of the surrounding particles. This process is known as hindered settling and consists of various effects: flow and wake formation around the particles and the increase of density and viscosity of the suspension. Hindered settling effects with decreasing settling velocities are dominant for \(c > 5\) to 10 kg/m\(^3\).

The hindered settling effect was studied experimentally by Richardson and Zaki (1954) and Richardson and Meikle (1961) using glass-type particles (ballotini) with particle sizes in the range of 35 to 1000 \(\mu m\) and alumina powder with a particle size of about 5 \(\mu m\). They found that the hindered settling effects can be represented as: \(w_s = w_{s,max}(1-\phi)^n\) with \(\phi = \) volume concentration, \(w_{s,max} = \) maximum settling velocity (input value). The \(n\)-coefficient varies in the range \(n = 2\) to 4.

Herein, the hindered settling factor is represented, as:

\[
w_s = w_{s,max}(1-\phi/\phi_{soil})^2
\]

(5.15)

with: \(\phi = \) volume concentration, \(\phi_{soil} = \) volume concentration at soil structure conditions (formation of soil network structure; \(\phi_{soil} = 0.05\) to 0.15).

The effect of the concentration on the fluid viscosity is herein neglected as it is assumed that this effect is implicitly represented by the empirical hindered settling function.

![Figure 5.5: Settling velocity as function of mud concentration](image-url)
5.4.2 Initial self-weight consolidation processes

At the end of the hindered settling process, the mud particles and flocs are in direct contact with each other and a primary stage of consolidation is initiated with seepage flows of water through the pores between the particle-floc skeleton structure. The mud concentration at the initiation of this self-weight consolidation process is known as the soil concentration ($\phi_{soil}$). The consolidation process will go on until the end concentration of the primary consolidation process is reached, which is the start of the secondary (Terzaghi) consolidation stage. The primary consolidation process is accompanied by relatively large strains in contrast to the secondary consolidation stage with relatively small strains. The settlement rate of the mud interface during the primary consolidation process can be determined by using a macro-scale approach for the total mud height, see Figure 5.6.

The volume (with area A) of the mud suspension with height $h_{mud}$ is:

$$V_{mud} = V_{sediment} + V_{water} = A (1-\eta) h_{mud} + \eta A h_{mud} = A h_{mud}$$

There is a volume change in time due to the outflow of seepage water through the pores, which yields:

$$\frac{dV_{mud}}{dt} = \frac{d(Ah_{mud})}{dt} = v_s A$$

$$\frac{dh_{mud}}{t} = v_s$$

$$\frac{dh_{mud}}{t} = w_{s,effective} = v_s = \alpha_k k$$

(5.16)

(5.17)

with:

$v_s = k (\rho_w g)^{-1} \frac{dp}{dx} = \alpha_k k = \text{Darcy seepage velocity (m/s)}$;

$\alpha_k = (\rho_w g)^{-1} \frac{dp}{dx} = \text{pressure gradient coefficient (range of 0.1 to 0.5), } \rho_w = \text{water overpressure (N/m}^2)$;

$k = \text{permeability coefficient (m/s)}$;

$\eta = \text{porosity coefficient; } t=\text{time; } h_{mud}=\text{mud height.}$

![Figure 5.6 Primary consolidation process](image)

Equation (5.17) states that the settlement rate of the mud interface is equal to $\alpha_k k$. The $\alpha_k$-coefficient representing the effect of the water overpressure gradient will gradually decrease to zero due to dewatering processes. The $\alpha_k$-coefficient is not explicitly computed, but is included in the $k$-coefficient which is assumed to a function of the mud concentration. The $k$-coefficient is assumed to decrease gradually to a small value for increasing mud concentration, see Figure 5.7.
The permeability coefficient is assumed to be a function of the mud concentration (Figure 5.7) as follows:

\[ k = k_{\text{max}} (1 - \phi_{\text{mean}})^n \]  

(5.18)

with: \( k_{\text{max}} \) = maximum permeability (\( \approx 10^{-6} \) to \( 10^{-7} \) m/s); \( n \) = coefficient (range of 5 to 10).

If dewatering can take place in two vertical directions (upward and downward), it is recommended to increase the permeability coefficient.

Using Equations (5.15) and (5.18), the effective settling velocity is described by (see Figure 5.7):

\[ w_{s,\text{effective}} = \begin{cases} w_{s,\text{max}} (1 - \phi_{\text{mean}}/\phi_{\text{soil}})^2 + k & \text{for } \phi_{\text{mean}} < \phi_{\text{soil}} \\ k & \text{for } \phi_{\text{mean}} > \phi_{\text{soil}} \end{cases} \]  

(5.19)

\[ k = k_{\text{max}} (1 - \phi_{\text{mean}})^n \]

Figure 5.7  Settling velocity and permeability as function of volume concentration

The parameters: \( w_{s,\text{max}}, k_{\text{max}}, \phi_{\text{soil}}, \) and \( n \) are input parameters; the \( \alpha_2 \)-coefficient is assumed to be 1; the parameter \( \phi_{\text{mean}} \) is the computed layer-averaged volume mud concentration as function of time. These parameters can be estimated from a simple laboratory consolidation experiment (initial mud height \( \approx 1 \) m).

5.4.3 Simplified self-weight consolidation model for soft mud soils

Winterwerp (1999) has proposed a numerical model for self-weight consolidation based on the one-dimensional continuity equation for sediments.

Based on this work, Van Rijn has formulated a more simplified self-weight consolidation model (MUDCONSOL.xls), which is explained hereafter.

The one-dimensional consolidation model MUDCONSOL.xls for fine sediments < 62 \( \mu \)m is based on the vertical continuity equation of sediments, as follows:

\[ \frac{\partial \phi}{\partial t} - \frac{\partial (w_{s,\text{eff}} \phi)}{\partial z} - \frac{\partial (D \partial \phi / \partial z)}{\partial z} = 0 \]  

(5.20)

The second term is the vertical settling term. The third term is the diffusion term, which yields upward sediment transport in the case of an exponentially increasing sediment concentration in downward direction (\( \partial^2 \phi / \partial z^2 = 0 \) for constant of linear concentration increase).
Equation (5.20) can be discretized as:

$$\phi_{i,t+\Delta t} = \phi_{i,t} + (\Delta t/\Delta z) w_{\text{eff},z} [\phi_{i+1,t} - \phi_{i,t}] + D \left( \Delta t/\Delta z^2 \right) [\phi_{i+1,t} - 2\phi_{i,t} + \phi_{i-1,t}]$$  \hspace{1cm} (5.21)

with:

- $\phi_{i,t} = c/\rho_s$ = volume concentration in point i at time t (-);
- $c$ = mass concentration ($=\rho_{\text{dry}}$, dry bulk density), (kg/m$^3$);
- $\rho_w$ = water density (kg/m$^3$);
- $\rho_s$ = sediment density ($=2650$ kg/m$^3$);
- $w_{\text{eff}}$ = effective settling velocity of mud particles (function of concentration), (m/s);
- $D$ = diffusion coefficient (m$^2$/s);
- $z$ = vertical coordinate (m).

Boundary conditions: $\phi_{i,t}$ at time t=0 for all z (initial uniform concentration; input value)

Boundary condition at bottom z=0: $\phi_{i,0,t+\Delta t} = \phi_{i,0,t} + (\Delta t/\Delta z) w_{\text{eff},z} [\phi_{i+1,0,t} - \phi_{i,0,t}]$

Boundary condition at surface $z=h$: $\phi_{i,h,t+\Delta t} = \phi_{i,h,t} + (\Delta t/\Delta z) w_{\text{eff},z} [\phi_{i+1,h,t} - \phi_{i,h,t}]$

Another boundary condition at z=0 is: $w_{\text{eff}} c_{z=0}^2 + D \partial \phi_{z=0}/\partial z$ or $c_{z=0} = c_{z=0+\Delta z}/[1-(w_{\text{eff}}/D) \Delta z]$.

This latter condition means that the downward settling term is equal to the upward diffusive term (local equilibrium at the bottom without deposition). Numerical solution requires very small grid sizes near the bed to get a stable solution and is, therefore, not used herein.

The settling velocity is given by Equation (5.19):

$$w_{\text{effective}} = w_{\text{max}} (1-\phi_{\text{mean}}/\phi_{\text{soil}})^2 \quad \text{for} \quad \phi_{\text{mean}} < \phi_{\text{soil}}$$  \hspace{1cm} (5.22)

$$w_{\text{effective}} = k_{\text{max}} (1-\phi_{\text{mean}})^n \quad \text{for} \quad \phi_{\text{mean}} > \phi_{\text{soil}}$$

with:

- $\phi_{\text{mean}}$ = depth-mean volume concentration of the lower layer with concentrations $>\phi_0$;
- $\phi_{\text{soil}}$ = $c_{\text{soil}}/\rho_s$ = volume concentration at the onset of primary consolidation phase (-);
- $c_{\text{soil}}$ = mass concentration at the onset of primary consolidation phase (input value 300 to 400 kg/m$^3$);
- $\phi_0$ = initial volume concentration at t=0 (input value);
- h = height of water column (-), (input value);
- $k_{\text{max}}$ = permeability coefficient (input value);
- n = coefficient (=5 to 10; input value).

The total sediment mass (kg/m$^2$) in the water column is: $M_s = \rho_s c \Delta z$.

Using an equidistant grid ($\Delta z = 0.01 h$; 101 grid points over thickness h), this yields:

$$M_{s,t} = \sum c_{i,t} \Delta z = 0.5 c_{1,t} + 0.5 c_{N,t} + \sum_{i=2}^{N-1} c_{i,t}$$  \hspace{1cm} (5.23)

Deposition: $\Delta M_{s,t} = M_{s,t+1} - M_{s,t}$

with: $c_{1,t}$ = mass concentration in point 1 at bottom, $c_{N,t}$ = mass concentration in point N at surface.

It is assumed that the potential deposition quantity $\Delta M_{s,t}$ remains in suspension in the lower layer (with volume concentrations >0.001). The potential deposition is redistributed as a uniform concentration over the lower layer. This effect reduces the effective settling rate of the mud interface.
5.4.4 Example and calibration computations

Settling column experiment; initial mud height=0.36 m
The spreadsheet model MUDCONSOL.xls (based on Equations (5.20) to (5.23)) has been used to compute the time development of the mud concentration profiles of a laboratory consolidation experiment in a settling column of 0.36 m (Van Rijn, 2017), see Figure 5.8. Six experiments have been done simultaneously with initial concentrations of $c_o=15$ to $300$ kg/m$^3$. Herein, the results of one settling column with $c_o= 200$ kg/m$^3$ are used (second column from the left). The mud was taken from the harbour basin of Noordpolderzijl (The Netherlands). The mud consisted of 35% clay/lutum < 4 μm, 30% silt of 4-62 μm and 35% fine sand > 62 μm. The $d_{50}$ of the mud sample was 15 μm. The maximum mud floc settling velocity was determined to be about 1.5 mm/s at a mud concentration of about 2 kg/m$^3$ (in saline seawater).

An initial uniform mud concentration of $c_o= 200$ kg/m$^3$ was generated in the perspex column by stirring/mixing with a simple wooden stick with a small perforated footplate. The height of the mud interface was measured over time (about 1 week), see Figure 5.9. The input data are given in Table 5.3.

Figure 5.9 shows the measured and computed values of the relative mud height ($h_{mud}/h_o$). The parameters $w_{s,max}$ and $k_{max}$ were varied to obtain the best agreement (see input Table 5.3). The soil concentration was set to 400 kg/m$^3$. The test results clearly show the presence of the relatively fast hindered settling process (< 0.1 day) followed by the much slower primary consolidation process between 0.1 and 7 days.

Figure 5.10 shows two computed concentration profiles after 2 hours and after 5 days. A clear mud interface can be observed. The total mud mass in the column is 72 kg/m$^2$ (0.36x200) and remains constant in time.

<table>
<thead>
<tr>
<th>Layer thickness</th>
<th>h</th>
<th>0.36 (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum settling velocity (flocculated)</td>
<td>Ws,max</td>
<td>0.001 (m/s)</td>
</tr>
<tr>
<td>Sediment density</td>
<td>Rhos</td>
<td>2650 (kg/m$^3$)</td>
</tr>
<tr>
<td>Maximum Permeability (at gel concentration)</td>
<td>Kmax</td>
<td>1.00E-06 (m/s)</td>
</tr>
<tr>
<td>Water density</td>
<td>Rhow</td>
<td>1020 (kg/m$^3$)</td>
</tr>
<tr>
<td>Soil concentration</td>
<td>Csoil</td>
<td>400 (kg/m$^3$)</td>
</tr>
<tr>
<td>Concentration at time $t=0$</td>
<td>Ct=0</td>
<td>200 (kg/m$^3$)</td>
</tr>
<tr>
<td>Coefficient related to permeability</td>
<td>n</td>
<td>10 (-)</td>
</tr>
<tr>
<td>Diffusion coefficient</td>
<td>Do</td>
<td>0.000E+00 (m$^2$/s)</td>
</tr>
<tr>
<td>Time step hindered settling process</td>
<td>dt1</td>
<td>1 (minutes)</td>
</tr>
<tr>
<td>Time step primary consolidation process</td>
<td>dt2</td>
<td>0.007 (days)</td>
</tr>
</tbody>
</table>

Table 5.3 Input data laboratory settling experiment (initial height $h_o=0.36$ m); MUDCONSOL.xls
Figure 5.8  Consolidation columns (initial settling height= 0.36 m)

Figure 5.9  Relative mud height ($h_{mud}/h_o$) as function of time; relative mud height; $h_o$=0.36 m; $c_o$=200 kg/m$^3$

Figure 5.10  Mud concentration profiles at various times; $h_o$=0.36 m; $c_o$=200 kg/m$^3$
Settling column experiment; initial mud height=1.85 m

The spreadsheet model MUDCONSOL.xls (based on Equations (5.20) to (5.23)) has been used to compute the time development of the mud concentration profiles of a laboratory consolidation experiment with Delfzijl-mud (Test 2E) in a settling column of 1.85 m (Van Rijn, 2018). The initial concentrations is $c_0=300$ kg/m$^3$. The Delfzijl-mud consists of 80% fines ($< 62$ μm) and 20% sand. The input data are given in Table 5.4.

<table>
<thead>
<tr>
<th>Layer thickness</th>
<th>h</th>
<th>1.85 (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum settling velocity (flocculated)</td>
<td>$W_{s,\text{max}}$</td>
<td>0.0002 (m/s)</td>
</tr>
<tr>
<td>Sediment density</td>
<td>$\rho_h$</td>
<td>2650 (kg/m$^3$)</td>
</tr>
<tr>
<td>Maximum Permeability (at gel concentration)</td>
<td>$k_{\text{max}}$</td>
<td>3.50E-07 (m/s)</td>
</tr>
<tr>
<td>Water density</td>
<td>$\rho_w$</td>
<td>1020 (kg/m$^3$)</td>
</tr>
<tr>
<td>Soil concentration</td>
<td>$C_{\text{soil}}$</td>
<td>400 (kg/m$^3$)</td>
</tr>
<tr>
<td>Concentration at time $t=0$</td>
<td>$C_t=0$</td>
<td>300 (kg/m$^3$)</td>
</tr>
<tr>
<td>Coefficient related to Permeability</td>
<td>$n$</td>
<td>10 (-)</td>
</tr>
<tr>
<td>Diffusion coefficient</td>
<td>$D_0$</td>
<td>0.000E+00 (m$^2$/s)</td>
</tr>
<tr>
<td>Time step hindered settling process</td>
<td>$dt_1$</td>
<td>20 (minutes)</td>
</tr>
<tr>
<td>Time step primary consolidation process</td>
<td>$dt_2$</td>
<td>0.6 (days)</td>
</tr>
</tbody>
</table>

Table 5.4 Input data laboratory settling experiment ($h_0=1.85$ m; $c_0=300$ kg/m$^3$; D-mud); MUDCONSOL.xls

Figure 5.11 shows the measured and computed values of the dry density as function of time. The parameters $W_{s,\text{max}}$ and $k_{\text{max}}$ were varied to obtain the best agreement (see input Table 5.4). The soil concentration was set to 400 kg/m$^3$.

Figure 5.12 shows the computed density profiles at various times.

Figure 5.13 shows the computed settling velocity and permeability as function of time.

Figure 5.11 Measured and computed dry density as function of time; $h_0=1.85$ m; $c_0=300$ kg/m$^3$; D-mud
5.5 Simple empirical consolidation model for soft mud soils

5.5.1 Single mud layer

The dry mass density \( c \) of consolidated soil under water consisting of organic matter, clay/lutum, silt and sand with a layer thickness of 1 to 5 m can be described by (Van Rijn, 1993, 2006):

\[
c_{\text{end}} = 500 \left(1 + \alpha_b\right) f_{\text{org}} + 1200 f_{\text{silt}} + 1550 f_{\text{sand}}
\]  

(5.24)

in which: \( c \) = dry end density \( (\text{kg/m}^3) \), \( f = \text{fraction size (} \sum f = 1 \) ), \( f_{\text{org}} = \text{fraction of organic material (} 0 \) to 0.2), \( f_{\text{clay}} = \text{fraction of clay (} 0 \) to 0.5), \( f_{\text{silt}} = \text{fraction of silt}, f_{\text{sand}} = \text{fraction of sand, } \alpha_b = \text{coefficient upper load (} 0.1 \) to 0.3 depending on the layer thickness of sand or silt).

For example: \( \alpha_b = 0, f_{\text{org}} = 0.2, f_{\text{clay}} = 0.4, f_{\text{silt}} = 0.3, f_{\text{sand}} = 0.1 \) gives: \( c_{\text{end}} = 160 + 360 + 155 = 675 \text{ kg/m}^3 \).
The wet density is:

\[ \rho_{\text{wet}} = \rho_{\text{water}} + \left( (\rho_s - \rho_{\text{water}}) / \rho_s \right) c_{\text{dry}} = 1000 + 0.62 \times 675 = 1420 \text{ kg/m}^3 \]  

(5.25)

**Figure 5.14** shows Equation (5.25) for \( \rho_{\text{water}} = 1000 \text{ kg/m}^3 \) and \( \rho_s = 2650 \text{ kg/m}^3 \).

![Dry and wet density graph](image)

**Figure 5.14  Dry and wet density**

The consolidation time \( T_{\text{end}} \) to reach 95% of the final (end) density at the end of the secondary consolidation process, can be estimated with:

\[ T_{\text{end}} = 0.5 \rho_{\text{water}} g m_v h^2 / k \]  

(5.26)

where: \( h \) = layer thickness (m), \( g = 9.81 \text{ m/s}^2 \), \( k = \text{permeability coefficient} \approx 10^{-10} \text{ m/s} \) for muddy soils, \( m_v = \text{compressibility coefficient} \approx 10^{-5} \text{ for soft soils} \).

For example: \( h = 1 \text{ m} \), \( k = 10^{-10} \text{ m/s} \), \( m_v = 10^{-5} \text{ m}^2/\text{N} \), \( \rho_{\text{water}} = 1000 \text{ kg/m}^3 \) gives: \( T_{\text{end}} = 0.5 \times 10^9 \text{ s} \approx 15 \text{ years} \).

The behaviour of the density in time from the initial dry density \( (c_0) \) at \( t = 0 \) to the final dry density \( (c_{\text{end}}) \) at \( t = T_{\text{end}} \) can be described with an exponential (logarithmic) function:

- dry density: \( c_t = c + (c_{\text{end}} - c_t) \left( t/T_{\text{end}} \right)^\alpha \)  
- wet density: \( \rho_t = \rho_o + (\rho_{\text{end}} - \rho_o) \left( t/T_{\text{end}} \right)^\alpha \)

(5.27-5.28)

Based on the measured behaviour of Bangkok-mud (see **Figure 5.2**), the exponent is determined on \( \alpha = 0.4 \).

The height \( h_t \) at time \( t \) of a mud column with an initial height \( h_o \) can be described with (continuity):

\[ h_t = (c_o/c_t) h_o = h_o \left[ 1 + \left( (c_{\text{end}} - c_o) / c_o \right) \left( t/T_{\text{end}} \right)^{0.4} \right]^{-1} \]  

(5.29)

The settlement is: \( s_t = \Delta h_t = h_o - h_t = h \left( 1 - c_o/c_t \right) \)

Using the above equations, yields:

\[ s_t = \Delta h_t = h_o \left[ 1 - \left( 1 + \left( (c_{\text{end}} - c_o) / c_o \right) \left( t/T_{\text{end}} \right)^{0.4} \right) \right]^{-1} \]  

(5.30)

Equations (5.24) to (5.31) are implemented in the spreadsheet model MUDCONSOL.xls.
Example 1
\(c_o = 200 \text{ kg/m}^3\), \(c_{end} = 500 \text{ kg/m}^3\) and \(t = 0.25 \text{ T}_{end}\):  
\[\Delta h_{t=0.25T_{end}} / h_o = 1 - (1 + (1.5)(0.25)^0.4)^{-1} = 1 - (1.86)^{-1} \approx 0.45\]
So, at a quarter of the end time the settlement is about 45% of the initial height.
The relative end settlement is:  
\[\Delta h_{T_{end}} / h_o = 1 - (1 + 1.5)^{-1} = 1 - (2.5)^{-1} \approx 0.6 \text{ or } 60\% \text{ of the initial height.}\]
So, at a quarter of the end time the settlement already is 0.45/0.6 = 75% of the total settlement.

Example 2
\(c_o = 250 \text{ kg/m}^3\), \(c_{end} = 500 \text{ kg/m}^3\); \(h_o = 2.5 \text{ m}\); \(T_{end} = 10 \text{ years}\).
The consolidation starts after 90 days (construction period).
After 1 year (365 days), a sand layer (upper load) is placed on top of the mud layer to increase the consolidation process. The end density is assumed to increase to: \(c_{end} = 600 \text{ kg/m}^3\).
The initial density after 1 year is: \(c_{o,1 \text{ year}} = 340 \text{ kg/m}^3\), see Figure 5.15.

Figure 5.15 Settlement and volume density as a function of time; single mud layer of 2.5 m

Figure 5.15 shows the settlement of a mud layer over 10 years without and with upper load. The initial dry density is set to 250 \(\text{ kg/m}^3\). The total settlement of a mud layer with initial thickness of 2.5 m is approximately 1.3 m after 10 years (without upper load). The end height is 1.2 m. The end dry density is assumed to be 500 \(\text{ kg/m}^3\) after 10 years. The effect of an upper load (sand layer placed after 1 year) is also shown resulting in an additional settlement of about 0.2 m. The end dry density with upper load is assumed to be 600 \(\text{ kg/m}^3\).

5.5.2 Multiple mud and sand layers

To obtain a total mud layer package of about 5 m (final stage), the filling and consolidation of the soil layers in the mud compartment should be carried out in steps, as follows:
- layer of soft fluid mud of about 2.5 m;
- drainage layer of fine sand of 1 m;
- layer of soft fluid mud of 2.5 m;
- drainage layer of sand 1 m, etc.

This package can consolidate in about 3 to 5 years to a package of about 5 m thick due to the presence of horizontal drainage layers of sand (IJburg trial Smits 1998; Dankers 2006 and Torfs et al. 1996). If necessary,
drainage columns of sand can be made. Each thin drainage layer of sand should be placed after 3 months on top of the mud layer to be ensured that the mud layer already has a certain structure (matrix).

**Example 1**

Three mud layers are placed:
- Lower mud layer= 2.5 m; initial dry density= 250 kg/m$^3$; end density= 550 kg/m$^3$
- Middle mud layer= 3 m; initial dry density= 250 kg/m$^3$; end density= 450/500 kg/m$^3$
- Upper mud layer= 2 m; initial dry density= 250 kg/m$^3$; end density= 500 kg/m$^3$

Figure 5.16 shows the consolidation results after 10 years. The total soil package of 8.75 m consolidates to a value of 4.3 m after 10 years (height reduction of about 50%).

**Figure 5.16** Settlement as a function of time; package of three mud and two sand layers
5.6 Terzaghi consolidation model for firm soils

5.6.1 Consolidation processes

The consolidation process strongly depends on the time development of the soil, pore water and grain skeleton stresses. Soil stresses ($\sigma$) consist of pore water pressures ($p$) and sediment grain skeleton stresses ($\sigma_s$), as shown in Figure 5.17, resulting in: $\sigma_z = p_z + \sigma_{s,z}$ with $p_z$=static pore pressure, $p_0$=excess pore pressure ($\geq 0$ in long term equilibrium conditions without external load), $\sigma_{s,0,z}$= (original) grain stress before loading and $\sigma_{s,E,z}$ = excess grain stress ($\geq 0$ without external load), $z$= vertical coordinate (positive upward, $z=0$= base of layer). The soil stress is $\sigma_z=\rho_{\text{soil}} g (h-z)$, $h$= layer thickness.

When an external load ($q$ in N/m$^2$) is applied (sand body on terrain surface), the soil stress increases with a value $q$ giving: $\sigma_z=p_{\text{soil}} g (h-z) + q = p_{0,z} + p_{E,z} + \sigma_{s,0,z} + \sigma_{s,E,z}$ and $p_{E,z} + \sigma_{s,E,z} = q$. Hence, the external load is taken by the excess pore pressure and the excess grain stress. In permeable soil, a seepage flow is generated (vertical pore pressure gradient) to release the excess pore pressure and the load will on the long term be taken by the excess grain stress only.

![Figure 5.17 Soil, pore water and sediment skeleton stresses](image)

Consolidation is the process of soil compaction with closer packing of the sediment particles due to expulsion of pore water (dewatering) under the action of an external load.

Immediately after external loading, the compaction is small and the load is almost completely supported by excess pore water pressures (overpressure). Groundwater flow (seepage flow) is generated by the excess pore pressure until the excess pressure is returned to the static water pressure and the load is fully supported by sediment skeleton stresses.

Figure 5.18 shows the consolidation of sand and clay type soils. The consolidation of sand proceeds relatively rapid; the consolidation can be seen as an initial effect.

![Figure 5.18 Consolidation of sand and clay based on laboratory compression tests](image)
Clay-type soils show a complicated consolidation behaviour consisting of 3 phases:
1. initial deformation effect; the deformation often starts with a sudden jump followed by a gradual increase of deformation; the initial consolidation has been found to depend on $t^{0.5}$;
2. primary consolidation which can be described by the classical logarithmic (Terzaghi) consolidation;
3. secondary consolidation which is also known as creep or (secular effect of Keverling Buisman); continuing deformation after full decay of the excess pore water pressures; it can be described by a semi-logarithmic relation.

Soil compression tests (known as Oedometer tests) are executed in the laboratory by using external loads ($\sigma_{ex}$) at a small metal ring filled with soil, see Figure 5.18. A porous plate is present at the upper side or at both sides for dewatering. The external load is raised in steps. The consolidation coefficient can be determined from the test results. Various methods (Casagrande log-t method, Taylor $t^{0.5}$ method) are available to deal with the initial effect, which is often dominant because the sample layer thickness is relatively small (0.02 m). The consolidation coefficient can also be determined from field tests applying a (temporary) load by building a thick sand layer on the soil surface. Settlement markers (plates with long vertical rods) are placed on the subsoil surfaces. The vertical settlement of the rod ends are measured over time. The settlement behaviour can be simulated using a consolidation model by varying the consolidation coefficient until a good fit is obtained. Table 5.6 shows consolidation ($c_i$), permeability ($k$) and compressibility ($m_v$) coefficients for various soils.

<table>
<thead>
<tr>
<th>Type of soil</th>
<th>Wet bulk density $\rho_{oi}$ (kg/m$^3$)</th>
<th>Consolidation coefficient $c_i$ (m$^2$/s)</th>
<th>Compressibility coefficient $m_v$ (m$^3$/N)</th>
<th>Permeability coefficient $k$ (m/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Peat</td>
<td>1150</td>
<td>0.1-1 $10^{-8}$</td>
<td>100-1000 $10^{-7}$</td>
<td>0.01-10 $10^{-9}$</td>
</tr>
<tr>
<td>Clay</td>
<td>1450</td>
<td>0.1-1 $10^{-8}$</td>
<td>10-100 $10^{-7}$</td>
<td>0.01-1 $10^{-9}$</td>
</tr>
<tr>
<td>Silty clay</td>
<td>1600</td>
<td>1-10 $10^{-8}$</td>
<td>1-10 $10^{-7}$</td>
<td>1-100 $10^{-9}$</td>
</tr>
<tr>
<td>Sandy clay</td>
<td>1800</td>
<td>1-10 $10^{-8}$</td>
<td>1-10 $10^{-7}$</td>
<td>1-100 $10^{-9}$</td>
</tr>
</tbody>
</table>

$k=\rho_w g c_i m_v$ with $\rho_w$ = water density (kg/m$^3$), $g$ = 9.81 m/s$^2$

Table 5.6 Consolidation and permeability coefficients

Empirically, it has been found that during the primary consolidation phase the compression values $\varepsilon_1$ of a laboratory sample in the end stage due to external load $\sigma_{ex}$ and $\varepsilon$ due to external load $\sigma_{ex}$ (with $\varepsilon > \varepsilon_1$) load is raised in steps) can be described by the log-linear compression law of Terzaghi, as follows:

$$\varepsilon - \varepsilon_1 = \Delta \varepsilon = (1/C_{10}) \log(\sigma_{ex}/\sigma_{ex1}) = (1/C_{10}) \log(1 + \Delta \sigma_{ex}/\sigma_{ex1}) = (1/C_{10}) \ln(1 + \sigma_{ex}/\sigma_{ex1})$$

$$\Delta \varepsilon / \Delta \sigma_{ex} \cong 1 / (2.3 C_{10} \sigma_{ex1})$$

$$\Delta \sigma_{ex} / \Delta \varepsilon = E \cong 2.3 C_{10} \sigma_{ex1} \cong C \sigma_{ex1}$$

with: $\varepsilon = \Delta h/h =$ dimensionless compaction, $E =$ elastic modulus, $C = 2.3 C_{10}$. This suggests a linear elastic behaviour of soil, which works reasonably well in practice.

Usually, the voids ratio ($e$) is used in the anglo-saxon countries, as follows: $e_1 - e = \Delta e = C_e \log(\sigma_{ex}/\sigma_{ex1})$
with: $e =$ void ratio=$V_w/V_s=(V-V_s)/V_s$, $V =$ Volume, $V_w$=water volume, $V_s$=sediment volume, $C_e =$ compression index (from plot of $\Delta e$ against $\log(\sigma_{ex}/\sigma_{ex1})$).

The void ratio is a similar parameter as the porosity factor $\eta = e/(1+e)$.

The void ratio is related to the compression $\Delta e = \Delta V/V$ with: $V = (1+e)V_s$ $\Delta V = \Delta e V_s$ Using: $\Delta V/V = \Delta e$, it follows that: $\Delta e = \Delta e/(1+e)$
with: $V_s$=volume of sediment particles (constant)

This yields: $\Delta e = - [C_e/(1+e)] \log(\sigma_{ex}/\sigma_{ex1})$ and $C_e=(1+e)/C_{10}$
5.6.2 Terzaghi consolidation model

The consolidation of multiple firm soil layers can be computed by using numerical models (for example: D-SETTLEMENT of Deltares) based on the classical theory of Terzaghi. The classical theory of Terzaghi (soil mechanics, Verruijt 2001, 2012) is based on the following assumptions:

- increase of internal stresses due to presence of external load \( q \); the consolidation is caused by the excess stresses under external loading; static stresses are constant in time;
- self weight consolidation is not taking into account (only changes due to external loads \( q \));
- small deformations; firm saturated soil; water table level above upper interface; \( z \) is positive in upward direction (\( z=0 \) at base of layer);
- soil permeability \( k \) is constant in vertical (and horizontal) direction;
- soil properties are uniform in horizontal direction (lateral);
- soil porosity is almost constant in vertical direction;
- water is not compressible;
- soil layer with thickness \( h \) can dewater in one direction (impermeable lower interface; clay layer above rock layer) and in two directions (upper interface and lower interface are permeable; clay layer between two sand layers; see Figure 5.19).

The spreadsheet model MUDCONSOL.xls is available for settlement computations of a single layer.

Two consolidating soil layers are distinguished (see Figure 5.19):
- A. clay layer with thickness \( h \) between two sand layers; dewatering in two directions;
- B. clay layer with thickness \( h \) resting on an impermeable lower layer; dewatering in one direction.

The stresses are:

\[
\sigma = p + \sigma_s = \text{soil stress at level } z \text{ above base layer};
\]
\[
\sigma = \text{soil stress } = p_{\text{top}} g a + p_{\text{layer}} g (h-z);
\]
\[
p_{\text{top}} = \text{wet bulk density of top layer above consolidating layer};
\]
\[
p_{\text{layer}} = \text{initial wet bulk density of consolidating layer with initial thickness } h;
\]
\[
h = \text{initial thickness of consolidating layer};
\]
\[
a = \text{thickness of top layer};
\]
\[
z = \text{height above base of consolidating layer};
\]
\[
p = p_{o} + p_{w} = \text{water pressure at level } z \text{ above base layer};
\]
\[
p_{o} = \text{static water pressure } = p_{w} n + p_{w} g (h-z);
\]

---

Figure 5.19 Case A: dewatering in one direction (left) and Case B: dewatering in two directions (right)
\( p_e \) = excess water pressure;
\( n \) = height of water table above upper interface of consolidating layer;
\( \sigma_s = \sigma_{s,E} + \sigma_{s,E} \) = skeleton grain stress at level \( z \) above lower interface of consolidating layer.
\( \sigma_{s,E} \) = skeleton grain stress before loading, \( \sigma_{s,E} \) = excess skeleton stress;
\( \partial \sigma / \partial t = 0 \): yields: \( \partial p / \partial t + \partial \sigma_s / \partial t = 0 \) and also \( \partial p_e / \partial t + \partial \sigma_{s,E} / \partial t = 0 \)

After loading with the external load, it is valid that: \( p_e + \sigma_{s,E} = q \)
The load is taken by the sum of the excess pore water pressure and the excess grain stress.
Initially, \( \sigma_{s,E} = 0 \) and \( p_e = q \); finally after complete dewatering: \( \sigma_{s,E} = q \) and \( p_e = 0 \);

**Deformation in vertical direction**
The vertical deformation over length \( \Delta z \) is
\( \Delta u = u + (\partial u / \partial z) \Delta z - u = (\partial u / \partial z) \Delta z = \varepsilon_z \Delta z \)
with \( \varepsilon_z = \partial u / \partial z = \) vertical strain (dimensionless),
\( u = \) vertical displacement.
The total strain is \( \varepsilon_{tot} = \varepsilon_z + \varepsilon_x + \varepsilon_y \)

**Volume change due to compression**
It is postulated that the vertical strain due to an external load is given by a linear (elastic) expression:
\[ \varepsilon_z = -m_v \sigma_{s,E} \]
\[ \Delta \varepsilon_z = -m_v (\partial \sigma_{s,E} / \partial t) \Delta t = m_v (\partial p_e / \partial t) \Delta t \] (5.32)
with: \( m_v = \) compressibility coefficient \( (m^2/N = ms^2/kg) \); negative sign for compression \( (z \) is positive upward).

**Ground water flow due to compression**
The net volume of water (inflow minus outflow) leaving per unit time in vertical direction through the horizontal plane of volume with sides \( \Delta x, \Delta y \) and \( \Delta z \) is given by:
\[ \Delta(\Delta V) = dv \Delta x \Delta y \Delta t = (dv/dz) \Delta x \Delta y \Delta z \Delta t \]
\[ \Delta \varepsilon_z = \Delta(\Delta V)/(\Delta x \Delta y \Delta z) = (dv/dz) \Delta t = [k/(\rho_w g)] \partial^2 p_e / \partial z^2 \Delta t \] (5.33)
with: \( v = \) seepage flow= \(-k/(\rho_w g)\) \( \partial p_e / \partial z \) and \( dv/dz=[k/(\rho_w g)] \partial^2 p_e / \partial z^2 \); upward (positive) flow for \( \partial p_e / \partial z < 0 \).
Thus:
Vertical strain due to compression: \( \Delta \varepsilon_z = -m_v \Delta \sigma_{s,E} = m_v (\partial p_e / \partial t) \Delta t \)
Outflow of groundwater due to compression: \( \Delta \varepsilon_z = -[k/(\rho_w g)] \partial^2 p_e / \partial z^2 \Delta t \)

**Terzaghi equation**
From Equations (5.32) and (5.33), it follows that:
\[ m_v (\partial p_e / \partial t) \Delta t - [k/(\rho_w g)] \partial^2 p_e / \partial z^2 \Delta t = 0 \]
\[ \partial p_e / \partial t - [k/(\rho_w m_v g)] \partial^2 p_e / \partial z^2 = 0 \]
\[ \partial p_e / \partial t - c_v \partial^2 p_e / \partial z^2 = 0 \] (5.34)
with: \( c_v = k/(\rho_w m_v g) \) = consolidation coefficient \( (m^2/s) \).
Equation (5.34) is a diffusion type of equation resulting in the gradual decay of the water excess pressure.
Equation (5.34) can also be expressed in terms of the grain stress as: \( \partial \sigma_{s,E} / \partial t - c_v \partial^2 \sigma_{s,E} / \partial z^2 = 0 \)
It is valid that: \( p_e + \sigma_{s,E} = q = \) external load=constant; \( \partial p_e / \partial t + \partial \sigma_{s,E} / \partial t = 0 \); \( \partial p_e / \partial z + \partial \sigma_{s,E} / \partial z = 0 \); \( \partial^2 p_e / \partial z^2 + \partial^2 \sigma_{s,E} / \partial z^2 = 0 \)

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The settlement at each level z is given by: \( \Delta s = \Delta \epsilon \), \( \Delta z = -m_v \sigma_{s,E} \Delta z \).
The total settlement is: \( s = \sum \Delta s, \Delta z \).
with: \( \sigma_{s,E} = q - p_t \) = grain stress; \( \epsilon = \Delta \epsilon / (1 + \epsilon) \) and \( \Delta \epsilon = (1 + \epsilon) \epsilon \).
Thus, the new void ratio is: \( e_{\text{old,new}} = e_{\text{old}} + \frac{(1 + \epsilon) \epsilon - e_{\text{old}}}{1 + \epsilon} \).
The new dry bulk density is given by: \( \rho_{\text{dry,new}} = [1 - \eta] \rho_s = [1 - \eta] / [1 + e_{\text{new}}] \rho_s \)
The wet bulk density is given by: \( \rho_{\text{wet,new}} = \rho_v + [\rho_s - \rho_v] / \rho_s \)
with: \( \eta = \text{porosity} = e / (1 + e) \), \( \rho_v = \text{water density} \), \( \rho_s = \text{sediment density} \).

**Case A: Dewatering in one direction (Figure 5.19)**
The boundary conditions are (see also Table 5.8):
At time \( t=0 \): \( p_t = q \) for all \( z \);
Upper interface \( z=h \): \( p_t =0 \) for all \( t \);
Lower interface \( z=0 \): \( p_t =0 \) for all \( t \).
The solution reads as: \( p_t = q \sum_{m=0}^{\infty} \left[ (4/\alpha) \sin(\alpha \pi z / h) \right] e^{\alpha t} \)
with:
\( \alpha = (2m+1)\pi = \text{coefficient (-)} \);
\( \beta = c_v \alpha^2 / h^2 = \text{coefficient (1/s)} \);
\( c_v \) = consolidation coefficient (m²/s).
The sine-term represents the vertical distribution over the layer thickness and the exponential term represents the time effect. Equation (5.35) can be simply computed in a spreadsheet model. The theoretical pore pressures at small \( t \)-values are not very accurate, as laboratory tests have shown that the settlement during the initial phase is related to \( t^{0.5} \) rather than to \( e^{\alpha t} \).
The pore pressure only depends on the external load \( q \); self-weight compaction is not taken into account. If the time \( t \) is sufficiently large, only the first term can be used, as follows: \( p_t = q \left[ (4/\pi) \sin(\pi z / h) \right] e^{\beta t} \)
with \( \beta = c_v \pi^2 / h^2 \).
The total settlement \( s \) over the layer thickness (h) can be computed from: \( \Delta s = \epsilon \sum \Delta s, \Delta z \) yielding \( s = \sum \Delta s, \Delta z \).
Using \( \epsilon = m_v \sigma_{s,E} \) and \( \sigma_{s,E} = q - p_t \), it follows that:
\begin{align*}
\Delta s & = \sum \Delta s, \Delta z = - \sum m_v q \cdot h - m_v p_t \Delta z = - \sum m_v q \cdot h - m_v p_t \Delta z \\
\text{The solution is:} & \quad s = -q \cdot h \cdot m_v \left[ 1 - m_v \sum_{m=0}^{\infty} \left[ (8/\alpha^2) \right] e^{\alpha t} \right] \\
\text{The time at which s is equal to 99% of the final settlement s}_{\text{end}} \text{ is used as the end time t}_{\text{end}}. \\
\text{Thus:} & \quad t = t_{\text{end}} \text{ and } s = 0.99 s_{\text{end}}
\end{align*}
An estimate is: \( t_{\text{end}} = 0.5 h^2 / c_v = 0.5 \rho_v g \cdot m_v h^2 / k \)
\begin{equation}
(5.38)
\end{equation}
The end settlement (compression; negative) can be determined by the first term only, yielding:
\( s = -q \cdot h \cdot m_v \left[ 1 - (8/\pi^2) e^{\alpha t} \right] \)
\begin{equation}
(5.39)
\end{equation}
Using: \( t = t_{\text{end}} = 0.5 h^2 / c_v \), an approximation expression can be determined: \( s_{\text{end}} = -0.99 q \cdot h \cdot m_v \)
\begin{equation}
(5.40)
\end{equation}
If the lower layer is impermeable (Case B), the solution of Case A can also be used by taking the layer thickness twice as large. The settlement is twice as large and has to be divided by a factor of 2.
**Figure 5.20** shows the time development of the porewater when an external load is applied at time \( t=0 \). Initially, the load causes a rise of excess pore water pressure yielding: \( p_t = q \) at all \( z \). For \( t>0 \), the excess pore water pressures will gradually decay to the original values and the excess grain stresses will increase.
**Figure 5.21** shows similar distributions for an example computation based on spreadsheet model MUDCONSOL.xls: clay layer \( h=5 \) m, water table at surface, external load of \( q=50000 \) N/m² (sand body with height of about 3 m), \( t_{\text{end}} = 0.5 h^2 / c_v = 150 \) days. Input data are given in Table 5.7. The pore pressure gradually reduces to zero. The total settlement is \( s_{\text{end}} = 0.25 \) m after 150 days. The wet density increases from the initial value of 1500 kg/m³ to 1502 kg/m³.
Figure 5.20  Pore pressures and grain stresses after loading (dewatering in two directions)

Figure 5.21  Left: Ratio of excess pore pressure and static pore pressure at $t=0.01T_{end}$, $t=0.3T_{end}$, $t=0.8T_{end}$
Right: Ratio of excess pore pressure and external load at $t=0.01T_{end}$, $t=0.3T_{end}$, $t=0.8T_{end}$
Case A: layer thickness $h=5$ m; $T_{end}=150$ days; lower layer is permeable
Figure 5.22 shows the settlement as function of time for two values of the $c_v$-coefficient. The end settlement is $s_{\text{end}} = -0.25$ m. The $c_v$-coefficient influences the end time. If the $c_v$-value is 10 times smaller the end time increases by a factor of 10 from 150 to 1500 days. The $c_v$-value does not affect the settlement; only the time scale. The settlement value $s$ is only affected by the $m_v$-coefficient.

![Settlement as function of time](image)

**Figure 5.22** Settlement as function of time

Layer thickness $h$ | $5$ (m)  
Thickness of top layer $a$ | $0$ (m)  
height water table above interface $n$ | $0$ (m)  
External load $q$ | $50000$ (N/m²)  
Water density $\rho_w$ | $1000$ (kg/m³)  
Sediment density $\rho_s$ | $2650$ (kg/m³)  
Wet bulk density of consolidating layer $\rho_{\text{soil}}$ | $1500$ (kg/m³)  
Wet bulk density of top layer $\rho_{\text{soil,top}}$ | $0$ (kg/m³)  
Consolidation coefficient $c_v$ | $9.65E-07$ (m²/s)  
Compressibility coefficient $m_v$ | $1.00E-06$ (m²/N)

time $t$ | $45$ (days)

Tend $12953367.88$ (sec)  
$149.923293$ (days)

$k$ permeability coefficient $k$ | $9.47E-09$ (m/s)  
porosity value $\rho_{\text{por}}$ | $0.696969697$ (-)  
Dry bulk density $\rho_{\text{dry}}$ | $803.030303$ (kg/m³)  
void ratio $e$ | $2.3$ (-)

Solution for dewatering at upper and lower interface; layer thickness $h$

Settlement at time $t$ (exact solution) | $-0.20381013$ (m)  
Settlement at Tend (approximation=0.99 × load × $h$ × $c_v$) | $-0.2475$ (m)

| Table 5.7 | **Input data of example computation; MUD-CONSOL.xls; Case A with permeable lower layer ($T_{\text{end}}=150$ days)** |

Equation (5.37) can be written as: $s/h = m_v q t'$ or as $\varepsilon = \left(\sigma/E\right) t'$, which is a special version of the elasticity law of Hooke representing the deformation of elastic bodies ($t' = $ time factor; $\sigma=q$, $E=1/m_v$).

The end settlement (compression; negative) can be determined by the first term only, yielding:

$$s = -q h m_v \left[1 - \left(8/\pi^2\right) e^{8t}\right]$$  \hspace{1cm} (5.41)

Using: $t=T_{\text{end}}=0.5h^2/c_v$, an approximation expression can be determined: $s_{\text{end}} \approx -0.99 q h m_v$  \hspace{1cm} (5.42)
The consolidation time $T_{\text{end}}$ is related to $h^2$, which means that the consolidation time $T_{\text{end}}$ increases by a factor of 9 for a layer of 3 m compared to a layer of 1 m. In the case of very thick layers, the final consolidation time will be very large (10 to 100 years).

The consolidation process can be accelerated by using vertical drains, consisting of:
- sand columns (diameter 0.2-0.3 m);
- artificial drainage strips/pipes (diameter= 0.03-0.05 m; covered with geofabrics) or strips (width=0.1 m; thickness=0.01-0.02 m; covered with geofabrics); vacuum pumping (underpressure) can be used to speed up the dewatering process, see Section 4.5.

Using vertical drains, horizontal dewatering flows are generated in the consolidating layer. The spacing of the drains is about $1/3$ to $1/2$ of the layer thickness.

Case B: dewatering in two directions (Figure 5.19)
The boundary conditions are (see also Table 5.8):
At time $t=0$: $p_E=q$ for all $z$;
Upper interface $z=h$: $p_E=0$ for all $t$;
Lower interface $z=0$: $\frac{\partial p_E}{\partial z}=0$ for all $t$.

The solution reads as: $p_E=q \left[ \frac{4}{\pi} \cos \left( \frac{0.5\mu z}{h} \right) e^{-\lambda t} \right]$ (5.43)
with:
- $\mu=(2m-1)\pi=$coefficient (-);
- $\lambda=0.25c_v \frac{\mu^2}{h^2}=$coefficient (1/s);
- $c_v=$ consolidation coefficient (m$^2$/s).

If the time $t$ is sufficiently large, only the first term can be used, as follows: $p_E=q \left[ \frac{4}{\pi} \sin \left( \frac{\pi z}{h} \right) e^{-\lambda t} \right]$ with $\lambda=0.25c_v \frac{\pi^2}{h^2}$.
The solution of the settlement is: $s=q h m_v \left[ 1 - \sum_{m=1}^{\infty} \frac{8}{\pi^2} e^{-\lambda t} \right]$ (5.44)
The end settlement (compression; negative) can be determined by the first term only, yielding:
$s=q h m_v \left[ 1 - \sum_{m=1}^{\infty} \frac{8}{\pi^2} e^{-\lambda t} \right]$ (5.45)
Using: $t=T_{\text{end}}=2h^2/c_v$, an approximation expression can be determined: $s_{\text{end}}=-0.99 q h m_v$ (5.46)

The total settlement is exactly the same as that for Case A, but the time scale is a factor 4 larger if the same layer thickness is used. Dewatering is only in vertical upward direction for Case B. So, the dewatering time is 4 times larger.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Case A Dewatering in two directions</th>
<th>Case B Dewatering in one direction</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pore pressure $p_E$ (N/m$^2$)</td>
<td>$p_E=q \sum_{\alpha} \frac{4}{\alpha} \sin(\alpha z/h) e^{\beta t}$</td>
<td>$p_E=q \sum_{\alpha} \frac{4}{\alpha} \cos(\alpha z/h) e^{\beta t}$</td>
</tr>
<tr>
<td>Settlement $s$ (m)</td>
<td>$s=0.5h^2/c_v$</td>
<td>$s=-0.99 q h m_v$</td>
</tr>
<tr>
<td>End time $T_{\text{end}}$ (s)</td>
<td>$T_{\text{end}}=2h^2/c_v$</td>
<td>$T_{\text{end}}=0.5h^2/c_v$</td>
</tr>
<tr>
<td>End settlement $s_{\text{end}}$ (m)</td>
<td>$s_{\text{end}}=-0.99 q h m_v$</td>
<td>$s_{\text{end}}=-0.99 q h m_v$</td>
</tr>
<tr>
<td>$\alpha$ (-)</td>
<td>$(2m+1)\pi$</td>
<td>$(2m-1)\pi$</td>
</tr>
<tr>
<td>$\beta$ (1/s)</td>
<td>$c_v \alpha^2/h^2$</td>
<td>$0.25c_v \mu^2/h^2$</td>
</tr>
<tr>
<td>$\mu$ (-)</td>
<td>$\frac{\mu^2}{h^2}$</td>
<td>$\frac{\mu^2}{h^2}$</td>
</tr>
<tr>
<td>$\lambda$ (1/s)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 5.8 Summary of formulations
Figure 5.23 shows the pore pressure distributions for an example computation: clay layer h=5 m, water table at surface, external load of q= 50000 N/m² (sand body with height of about 3 m), T_end=2h²/c_v=600 days. Input data are given in Table 5.7. The pore pressure gradually reduces to zero. The total settlement is s_end=-0.25 m after 600 days. The wet density increases from the initial value of 1500 kg/m³ to 1502 kg/m³. The pore pressures fot t=0.01 T_end (upper plot) are not very accurate (excess pore pressure/external load values > 1), because only 10 terms have been used.

**Figure 5.23**  Left: Ratio of excess pore pressure and static pore pressure at t=0.01T_end, t=0.3T_end, t=0.8T_end  
Right: Ratio of excess pore pressure and external load at t=0.01T_end, t=0.3T_end, t=0.8T_end  
Case B: layer thickness h= 5 m; T_end= 600 days; lower layer is impermeable
Coefficients
The coefficients $c_v$ and $m_v$ can be determined from laboratory samples (compression tests) with small thickness of 0.02 m. The consolidation process proceeds rapidly. The $c_v$-coefficient follows from the time development. It has been found that the time development relates to $t^{0.5}$ for small $t$-values.

Two methods are generally used:
Casagrande (log $t$-method):
$$c_v=0.2 \frac{h^2}{t_{50\%}}$$
Taylor ($t^{0.5}$-method):
$$c_v=0.852 \frac{h^2}{t_{90\%}}$$

The $m_v$-coefficient follows from the end settlement $s_\infty=m_v h q$ resulting in:
$$m_v=\frac{s_\infty}{h q},$$
with:
- $h=$ layer thickness of sample (m);
- $s_\infty=$ end settlement (m);
- $t_{50\%}=$ time at which settlement is 50% of end value, $t_{90\%}=$ time at which settlement is 90% of end value ($s_\infty$);
- $q=$ external load (N/m$^2$);
- $c_v=$ consolidation coefficient (m$^2$/s); see Table 5.6;
- $m_v=$ compressibility coefficient (m$^2$/N); see Table 5.6;
- $\rho_w=$ water density;
- $k=\rho_w g c_v m_v=$ permeability coefficient (m/s); see Table 5.6.

The permeability coefficient can also be determined from seepage flow tests in the laboratory. The results of both methods may show considerable differences.

Secondary effect (creep)
The secondary creep effect can be taken into account by an additional term, as follows:

$$s = s_{end, primary} + s_{secondary} = h q \left[ m_{v,p} + m_{v,s} \log(t/t_p) \right]$$

with:
- $s_{end, primary}=$ end settlement of primary consolidation effect (Terzaghi consolidation);
- $s_{secondary}=$ settlement due to creep effect;
- $m_{v,p}=$ primary compressibility coefficient (m$^2$/N);
- $m_{v,s}=$ secondary compressibility coefficient (m$^2$/N);
- $t=$ time (days); $t_p=$ characteristic time at which primary effect is completed ($t_p=1$ day for laboratory test).

It is assumed that the primary effect proceeds relatively rapid so that the time development of that process can be neglected compared to the long-term time development of the creep effect ($t\approx 10 t_p$).

A practical expression which is often used, is given by:

$$s = h \left[ (1/C_p) + (1/C_s) \right] \ln(\sigma_{s,2}/\sigma_{s,1})$$

with:
- $C_p=$ coefficient primary effect (5-10 for peat; 10-50 for clay and 50-500 for sand; larger values for firm soils);
- $C_s=$ coefficient secondary effect (50-100 for peat; 100-500 for clay and 500-5000 for sand);
- $\sigma_{s,1}=$ grain stress value due to additional load (one layer-averaged value for each layer);
- $\sigma_{s,2}=$ grain stress value due to original load (one layer-averaged value for each layer).

The grain stresses are determined from $\sigma_{s,1}=\sigma_{soil,p1}$; the pore water stresses in each layer are measured in boreholes. The external load is added to compute the grain stress $\sigma_{s,2}=\sigma_{s,1}+q$. If the external load (sand
body) has a certain finite width at the terrain surface, the load at the depth of the layer concerned should be reduced by assuming a spreading angle (usually 45°).

In Anglo-saxon countries, the following expression is used:

$$e_0 - e = \Delta e = C_c \log(\sigma/\sigma_1) + C_\alpha \log(t/t_0)$$  \hspace{1cm} (5.49)

with:

- $e_0$: void ratio; $e$: void ratio at time $t_0$;
- $\sigma_1$: load/stress at $t=t_0$ (original situation; in the case of an external load due to sand body);
- $C_c$: compression index primary effect (larger values for soft soils);
- $C_\alpha$: compression index secondary effect.

A practical procedure for the computation of the total settlement due to an external load (sand body with height $h_{sand}$) placed on the terrain surface, is as follows:

1. take undisturbed soil samples from each layer in a borehole;
2. measure pore pressure $p_i$ in each layer;
3. apply original soil stress situation at each sample in laboratory setup; use waiting time of 1 day;
4. apply external load/stresses at each sample in steps and measure settlement of each sample;
5. determine consolidation and compressibility coefficients of each sample;
6. use coefficients to compute the settlement $s_i$ of each layer $h_i$ and add all value to determine $s$.

**Example Compressibility test**

Weakly consolidated mud from a small harbour basin (Noordpolderzijl, The Netherlands) has been used in a compressibility (Oedometer) test in soil laboratory of Wiertsema (Tolbert; March 2017). The mud sample was taken (15 March 2017) from the mud container (filled in November 2016) after a consolidation period of three to four months. Excess water was siphoned off. The dry density of the sample was measured to be in the range of 690 to 720 kg/m$^3$ (wet bulk density of about 1450 kg/m$^3$).

A ring with thickness of about 20 mm was filled with mud and subjected to an Oedometer test under a series of external loads. The results based on the Terzaghi-analysis method are given in Table 5.9. The permeability is computed as: $k = \rho_w \cdot g \cdot \gamma' \cdot m_v$.

After 7 steps (of 40 hours each; loading up to 128 kpa = 128,000 N/m$^2$ or 12.8 ton/m$^2$ or 1.3 kg/cm$^2$), the total settlement is about 7.3 mm (about 35% of the initial value of 20 mm). The compressibility $m_v$-coefficient decreases by a factor of about 50 between step 1 and step 7. The $k$-value decreases strongly by a factor of 20 between step 1 and 7. As the sample is more compressed/compacted, the permeability ($k$-value) will decrease.

A separate permeability test with an external load of 3.5 Kpa (=3500 N/m$^2$) has also been done yielding a permeability value of $k=4 \times 10^{-10}$ m/s.

The TERZAGHI-consolidation model (Van Rijn, 2017) has been used to compute the end settlement of a layer with thickness of $h=1$ m with dry bulk density of 700 kg/m$^3$ under an external load of $q=100000$ N/m$^2$ (about 10 ton/m$^2$) using the expression;

$$s_{end} = -0.99 \cdot m_v \cdot h = -0.99 \times 3 \pm 1 \times 10^{-6} \times 100000 \times 1 \cong -0.2 \text{ to } -0.4 \text{ m}$$

$$s_{end}/h = 0.2 \text{ to } 0.4 \text{ or } 20\% \text{ to } 40\%.$$ 

The time after which this settlement is completed is given by the expression (dewatering in upward direction only):

$$T_{end} = 0.5 \cdot h^2/c_v = 0.5 \times 1^2/(11 \pm 4 \times 10^{-10}) \cong 3.5 \text{ to } 7 \times 10^8 \text{ s} = 4000 \text{ to } 8000 \text{ days (about 10 to 20 years).}$$
### Table 5.9 Oedometer test results; Sample 02 Noordpolderzijl; mud after 3-4 months of natural consolidation; dry density of 700 kg/m³ (wet density of 1450 kg/m³)

<table>
<thead>
<tr>
<th>Test steps</th>
<th>Settlement (mm)</th>
<th>$c_v$-coefficient (m²/s)</th>
<th>$m_v$-coefficient (m²/N)</th>
<th>k-permeability (m/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 (0-2 kpa) 2500 minutes</td>
<td>20.00-18.06</td>
<td>log t-method</td>
<td>$5.2 \times 10^{-10}$</td>
<td>$45 \times 10^{-6}$</td>
</tr>
<tr>
<td>2 (2-3.6 kpa) 2500 minutes</td>
<td>18.05-17.32</td>
<td>log t-method $t^{0.5}$-method</td>
<td>$4.5 \times 10^{-10}$</td>
<td>$24 \times 10^{-6}$</td>
</tr>
<tr>
<td>3 (3.6-8.3 kpa) 2500 minutes</td>
<td>17.32-16.32</td>
<td>log t-method $t^{0.5}$-method</td>
<td>$5.5 \times 10^{-10}$</td>
<td>$11 \times 10^{-6}$</td>
</tr>
<tr>
<td>4 (8.3-16.2 kpa) 2500 minutes</td>
<td>16.32-15.43</td>
<td>log t-method $t^{0.5}$-method</td>
<td>$6.2 \times 10^{-10}$</td>
<td>$6.3 \times 10^{-6}$</td>
</tr>
<tr>
<td>5 (16.2-32 kpa) 2500 minutes</td>
<td>15.42-14.53</td>
<td>log t-method $t^{0.5}$-method</td>
<td>$7.9 \times 10^{-10}$</td>
<td>$3.2 \times 10^{-6}$</td>
</tr>
<tr>
<td>6 (32-63.6 kpa) 2500 minutes</td>
<td>14.53-13.60</td>
<td>log t-method $t^{0.5}$-method</td>
<td>$10.1 \times 10^{-10}$</td>
<td>$1.8 \times 10^{-6}$</td>
</tr>
<tr>
<td>7 (63.6-128.3 kpa) 2500 minutes</td>
<td>13.60-12.66</td>
<td>log t-method $t^{0.5}$-method</td>
<td>$12.9 \times 10^{-10}$</td>
<td>$0.9 \times 10^{-6}$</td>
</tr>
</tbody>
</table>

$\sum$ loads $= 2 + 3.6 + 8.3 + 16.2 + 32 + 63.6 + 128.3 = 254$ kpa

| total=7.33 | Weighted-average value of log t-method | $= (1/254) \times (2x5.2+3.6x4.5+8.3x5.5+16.2x6.2+32x7.9+63.6x10.1+128.3x12.9) \times 10^{-10}$ | $= 11 \times 10^{-10}$ |

| weighted-average value of log t-method | $= (1/254) \times (2x45+3.6x24+8.3x11+16.2x6.3+32x3.2+63.6x1.8+128.3x0.9) \times 10^{-6}$ | $= 3 \times 10^{-6}$ |

$k = \frac{\rho_w \cdot g \cdot c_v \cdot m_v}{\mu \cdot g \cdot c_v \cdot m_v}$

$= 1020 \times 9.81 \times 10^{-10}$

$= 11 \times 10^{-10} \times 3 \times 10^{-6}$

$= 0.33 \times 10^{-10}$

---

Note: Land reclamations of mud
Date: May 2019
6. Mud pollution during construction

6.1 General

Additional turbidity of the water by suspended mud during construction works can occur as a result of:
- dredging of sediment (mud);
- movement of the dredging equipment;
- mud spill during transportation (barges);
- dumping of sediment (mud) at the reclamation site.

Silt and mud with relatively low settling velocities can easily be transported over long distances. Turbulent processes in the water column will effectively reduce the settling velocities, resulting in additional horizontal spreading of fine sediments. The fine fraction < 4 μm will remain in suspension over a long period (days/weeks). The fraction from 4 to 32 μm will settle faster (within days) in the direct vicinity of the dredging/dumping site. Dredge plume measurements show that the fine sediment fractions can be spread over a maximum distance of 5 km. In windy conditions (winter) the plume concentrations will be rapidly mixed into the background concentrations of fines stirred up by wave action.

Based on various international studies of mud plumes caused by dredging processes (Van Rijn, 2005), the local increase of the fine sediment concentrations (within 100 m of the dredging site) can be up to 5000 mg/l at the bottom and 200 mg/l near the water surface. Generally, the mud concentrations go back rapidly to the background concentrations (within 1 day).

6.2 Theory of diffusion/dispersion/dilution processes

Dispersion refers to the spreading of mass as a bulk property (averaged concentrations) integrating all spreading/dispersion processes. Generally, the dispersion coefficient is larger than the turbulent mixing coefficient.

The one-dimensional advection-dispersion process of fine sediments in a horizontally uniform flow (dh/dx=0, du/dx=0) can be described by:

\[
\frac{\partial c}{\partial t} + u \frac{\partial c}{\partial x} - \varepsilon \frac{\partial^2 c}{\partial x^2} = 0
\] (6.1)

with: \(c\) = concentration, \(u\) = flow velocity (constant in space and time), \(\varepsilon\) = effective mixing coefficient (assumed to be constant in space and time).

Assuming a fluid at rest (\(u = 0\)), the expression becomes:

\[
\frac{\partial c}{\partial t} - \varepsilon \frac{\partial^2 c}{\partial x^2} = 0 
\] (6.2)

When a mass \(M\) (in kg/m²) is released at \(x = 0\) at time \(t = 0\), the solution of the one-dimensional diffusion equation is:

\[
c = \frac{M}{(4 \pi \varepsilon t)^{0.5}} \exp[-\frac{x}{(4 \varepsilon t)^{0.5}]} 
\] (6.3)

with: \(c\) = concentration (kg/m³), \(t\) = time, \(\varepsilon\) = constant diffusion/dispersion coefficient.

Continuity requires that: \(M = \int_{-\infty}^{\infty} c \, dx\). The solution represents a Gaussian distribution of the concentrations.

If the coordinate system is moving with the mean velocity \(u\), then Equation (6.3) representing a
symmetrical solution, is also valid with respect to the moving coordinate system.
The solution reads:

\[ c = \frac{M}{(4\pi \varepsilon t)^{0.5}} \exp[-\{(x'/\varepsilon t)^{0.5}\}^2] \] (6.4)

Defining \( x = ut + x' \) (see Figure 8.1) and thus \( x' = x - ut \), it follows that:

\[ c = \frac{M}{(4\pi \varepsilon t)^{0.5}} \exp[-\{(x-ut)/\varepsilon t^{0.5}\}^2] \] (6.5)

Equation (6.5) is shown in Figure 6.1 for M=10 kg/m\(^2\) (being a spike-type release at x= 0 at t= 0) and u= 0.5 m/s, \( \varepsilon = 0.1 \) m\(^2\)/s at t = 5, 20 and 100 seconds, showing the gradual spreading of the mass M in horizontal direction away from the source.
The maximum concentration (\( c_{\text{max}} \)) can be obtained for \( x=ut \) yielding: \( \exp[-\{(x-ut)/\varepsilon t^{0.5}\}^2] = 1 \).
The maximum concentration decreases in the downstream direction due to dispersion, Figure 6.1.
The maximum concentration in the 1D case decreases as: \( c_{\text{max}} \sim t^{-0.5} \).
The maximum concentration in the 2D and 3D case decreases as: \( c_{\text{max}} \sim t^{-1} \) and \( c_{\text{max}} \sim t^{-1.5} \).

Using this, the dilution factor (\( \gamma = c/c_0 \)) for 2D and 3D cases are given in Table 6.1.

<table>
<thead>
<tr>
<th>Time</th>
<th>Dilution factor ( \gamma )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>2D case</td>
</tr>
<tr>
<td>10 s</td>
<td>1/10</td>
</tr>
<tr>
<td>100 s</td>
<td>1/100</td>
</tr>
<tr>
<td>1000 s</td>
<td>1/1000</td>
</tr>
<tr>
<td>10000 s</td>
<td>1/10000</td>
</tr>
</tbody>
</table>

Table 6.1  Dilution factors for local source of (released) mud near the bottom

The time for fine particles (from a source near the bottom) to reach the water surface in a depth of 10 m due to mixing processes is of the order of 1000 s resulting in a dilution factor (\( \gamma = 1000^{1.5} = 1/30000 \)).
Thus: mud particles arriving at the water surface are diluted to almost nil.

Figure 6.2 shows the development of mud concentrations as function of time in vertical direction using \( u = w_s = -0.0002 \) m/s (advection is assumed to be equal to the settling velocity) and \( \varepsilon = \) mixing coefficient\( = 0.1 \) m\(^2\)/s. The mean concentration cloud at the source location (close the bed) will slowly sink to the bed due to the settling velocity, while the concentrations are mixed upwards by turbulence. In the case of a water depth of 100 m, the mud particles are estimated to arrive at the water surface after about 10000 s. The dilution factor of the mud concentration at the water surface is about 1/100 for this 1D vertical case, but will be much smaller due to additional mixing in both horizontal directions.
Figure 6.1  Dispersion/diffusion of concentration as function of x (horizontal) and t

Figure 6.2  Dispersion/diffusion of concentration as function of z (vertical) and t
7. References

Fossati, M. et al., 2007. Self-weight consolidation tests of the Rio de la Plata sediments. Instituto de Mecanica de los Fluidos e Ingenieria Ambiental (IMFIA), Uruguay
Van der Veen, c., 1962. Soil Mechanics (in Dutch), Kosmos, Amsterdam