EQUIVALENT ROUGHNESS OF ALLUVIAL BED

By Leo C. van Rijn

INTRODUCTION

The understanding and prediction of flow resistance in an alluvial channel with a movable bed is of considerable importance to hydraulic engineers. This subject, however, is complicated because the dimensions of the bed forms and, thus, the hydraulic roughness of the bed are dependent on the flow parameters (depth, velocity) and sediment properties (diameter, gradation). Therefore, first a relationship between the flow-sediment conditions and the dimensions of the bed forms must be established, while a relationship between the dimensions of the bed forms and their effective roughness needs also to be developed. In this note only the latter subject is considered for flow conditions with a plane (movable) bed or (movable) bed forms using a simple curve fitting approach. The prediction of movable bed forms as a function of flow-sediment conditions will be considered in a forthcoming paper.

PLANE BED

Usually the equivalent roughness of a plane bed is related to the largest particles of the bed material, $D_{90}, D_{50}, D_{40}$. The influence of the gradation, the shape of the particles, and the flow conditions are generally disregarded. In the literature the following values for the equivalent roughness of Nikuradse, $k_s$, can be found:

- Acker-White: $k_s = 1.25 D_{30}$
- Einstein: $k_s = D_{40}$
- Engelund-Hansen: $k_s = 2 D_{50}$
- Hey: $k_s = 3.5 D_{40}$
- Kamphuis: $k_s = 2.5 D_{90}$
- Mahmood: $k_s = 5.1 D_{30}$

To evaluate the preceding relationships, the writer has analyzed 120 sets of flume and field data (1,6,10,11) with plane bed conditions. Only flow data with a width-depth ratio larger than five were finally considered. Therefore, no side-wall correction was applied. The equivalent roughness of the plane bed was computed from the resistance equation for hydraulic rough flow:

Note.—Discussion open until March 1, 1983. To extend the closing date one month, a written request must be filed with the Manager of Technical and Professional Publications, ASCE. Manuscript was submitted for review for possible publication on January 9, 1981. This paper is part of the Journal of the Hydraulics Division, Proceedings of the American Society of Civil Engineers, ©ASCE, Vol. 108, No. HY10, October, 1982. ISSN 0044-796X/82/0010-1215/$01.00.
\[
\frac{\bar{u}}{u_*} = 2.5 \ln \left( \frac{R}{k_*} \right) + 6.23 \tag{7}
\]

in which \( \bar{u} \) = mean flow velocity (m/s); \( u_* \) = bed-shear velocity (m/s); and \( R \) = hydraulic radius (m).

For transitional flow \((5 < u_* k_*/v < 70, v = \text{kinematic viscosity coefficient})\), a White-Cooleybrook-type formula was used:

\[
\frac{\bar{u}}{u_*} = 2.5 \ln \left( \frac{R}{k_* + \frac{3.3 u_*}{v}} \right) + 6.23 \tag{8}
\]

The computed \( k_* \)-values were related to the \( D_{90} \) of the bed material and a particle mobility parameter, \( \Theta - \Theta_{cr} \), to investigate the influence of the transport mechanism \((\Theta = \tau_0/(\rho_0 - \rho) g D_{90} \tau_0 = \text{bed-shear stress}; g = \text{acceleration of gravity}; \rho_0 = \text{density of sediment}; \rho = \text{density of water}; \text{and } \Theta_{cr} = \text{critical mobility number according to Shields})\). As can be observed in Fig. 1, the scatter is large and the influence of the particle mobility number cannot be detected. The large \( k_* \)-values, which can be caused only by small irregularities in the bed, show that a completely plane bed does not exist. An average value which can be used is

\[k_* = 3 D_{90} \tag{9}\]

**Bed Forms**

The first authors who related the equivalent roughness (of Nikuradse) to the length, \( \lambda \), and the height, \( \Delta \), of the bed forms were Shinozuka and Tsubaki (12). Their results \((0.02 \leq \Delta/\lambda \leq 0.1)\) which are based on Eq. 7 can be represented by

\[k_* = 7.5 \left( \frac{\Delta}{\lambda} \right)^{0.57} \tag{10}\]

![Fig. 1.—Equivalent Roughness for a Plane Bed](image-url)
FIG. 2.—Equivalent Roughness for Bed Forms

For unidirectional flow in combination with waves, Swart (13) derived (0.05 ≤ Δ/λ ≤ 0.2):

\[
k_e = 25 \left( \frac{\Delta}{\lambda} \right)
\]

(11)

The influence of the grain roughness, which may be of interest for long, flat bed forms was neglected. To evaluate both relationships, the writer has analyzed 40 sets of flume and field data (1,9,11,12). Only flow data with a width-depth ratio larger than five and a depth-equivalent roughness ratio larger than 10 were considered. The latter condition is introduced because for relative large k_e-values the uniformity of the flow will be lost. The computed k_e-values can be represented by (0.01 ≤ Δ/λ ≤ 0.2):

\[
k_e = 1.1 \left( 1 - e^{-25 \Delta/\lambda} \right)
\]

(12)

This is shown in Fig. 2. Eq. 12 yields k_e-values which are considerably smaller than the relationships of Shinohara and Tsubaki, and Swart. This may be caused by the fact that Shinohara and Tsubaki, and Swart also used flume data with relatively large values of the k_e/R ratio. However, Eq. 7 is not valid for this type of data.

Klaassen (7) used Eqs. 7 and 12 to compute the flow depth for some 150 sets of flume and field data for which the dimensions of the bed forms were given (11). He found reasonable agreement between measured and computed values for a large range of flow conditions.

CONCLUSIONS

1. The effective or equivalent roughness of a plane movable bed varies from about 1–10 D_{50} of the bed material, and is not dependent on the transport stage.
2. The effective roughness of the bed forms is dependent on the height and the height-length (steepness) ratio of the (movable) bed forms.
APPENDIX.—REFERENCES


SIMPLIFIED TESTING OF HYDROLOGIC REGRESSION REGIONS

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INTRODUCTION

Hydrologists often use a regional regression model to transfer information collected at gaged sites to ungauged sites, or to improve estimates at gaged sites. Smaller standard errors of estimate can usually be obtained by subdividing the overall area into "homogeneous regions," if the number of gaged sites is ad-