The prediction of Bed-Forms and Alluvial Roughness

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ABSTRACT: A method is presented which enables the computation of the bed-form dimensions and the equivalent roughness of the bed forms in the lower, transitional and upper flow regime.

1 INTRODUCTION

To predict the flow depth and sediment transport rate of an alluvial channel, the hydraulic roughness of the movable bed surface should be known as a function of the flow conditions and sediment properties. Also the prediction of the bed-form dimensions is of importance to determine the allowable navigation depth in a channel with a movable bed surface.
Although various (empirical) methods are available to estimate the hydraulic roughness (Engelund-Hansen, 1967 and White et al, 1979), no reliable method is available to predict the bed-form dimensions. Therefore, the attention is focussed on the prediction of the bed-form dimensions while also the equivalent roughness of the bed forms is considered.

2 CHARACTERISTIC PARAMETERS

It is assumed that the dimensions of the bed forms are controlled by the bed-load transport rate and can be described by the same dimensionless parameters as used to describe the bed-load transport rate, being (van Rijn, 1981):

particle parameter

$$D_* = D_{50} \left[\frac{(s-1)\,g}{\nu^2} \right]^{\frac{1}{3}} \qquad (1)$$

in which: D_{50} = particle diameter of bed material, $s = \rho_s/\rho$ = specific density, ρ_s = sediment density, ρ = fluid density, g = acceleration of gravity and ν = kinematic viscosity coefficient.

transport stage parameter

$$T = \frac{(u'_*)^2 - (u_{*,cr})^2}{(u_{*,cr})^2} \qquad (2)$$

in which: $u'_* = (g^{0.5}/C')\,\bar{u}$ = bed-shear velocity related to grains, $\bar{u}$ = mean flow velocity, $C' = 18 \log(12\,R_b/3\,D_{90})$ = Chézy-coefficient related to grains, R_b = hydraulic radius of the bed, D_{90} = particle diameter of bed material, $u_{*,cr}$ = critical bed-shear velocity for initiation of motion according to Shields.

3 BED-FORM DIMENSIONS

Using the equation of continuity for the bed-load particles and a simple relationship for the bed-load transport rate [van Rijn, 1982], the relative bed-form height and steepness can be expressed as:

$$\frac{\Delta}{h} = F\left(\frac{D_{50}}{h}, D_*, T\right) \qquad (3)$$

$$\frac{\Delta}{\lambda} = F\left(\frac{D_{50}}{h}, D_*, T\right) \qquad (4)$$

in which: Δ = bed-form height, λ = bed-form length, h = flow depth.

To determine these functional relationships, a large quantity of experimental bed-form data were analyzed. In all, 84 flume data with particle diameters ranging from 190-2300 µm and 22 field data with particle diameters ranging from 490-3600 µm, were used [van Rijn, 1982].
The influence of the D_*-parameter was found to be negligibly small. The best agreement

between measured and computed results was obtained for:

$$\frac{\Delta}{h} = 0.11 \left[\frac{D_{50}}{h}\right]^{0.3} \left[1 - e^{-0.5\,T}\right] \left[25 - T\right] \quad (4)$$

$$\frac{\Delta}{\lambda} = 0.015 \left[\frac{D_{50}}{h}\right]^{0.3} \left[1 - e^{-0.5\,T}\right] \left[25 - T\right] \quad (5)$$

It is assumed that for $T \leq 0$ and $T \geq 25$ the bed is almost plane. Equation (4) and (5) as well as the error range of a factor 2, are shown in Figures 1 and 2. Nearly all data are within the plotted error range. From equations (4) and (5) it follows that:

$$\lambda = 7.3\ h \quad (6)$$

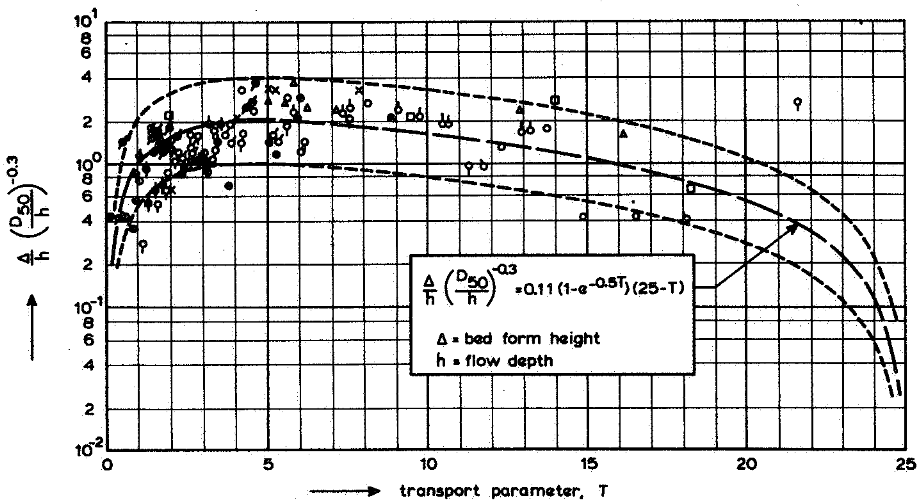

Fig. 1. Bed-form height as a function of particles size, flow depth and transport stage

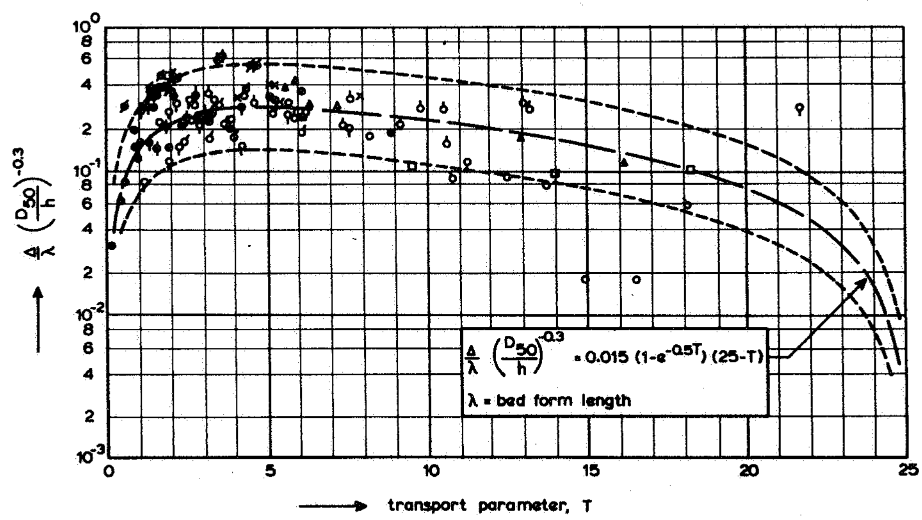

Fig. 2. Bed-form steepness as a function of particle size, flow depth and transport stage

4 EQUIVALENT ROUGHNESS OF BED FORMS

The hydraulic roughness of a movable bed surface is caused by grain roughness and form roughness, which can be expressed as:

$$k_s = F(D,\ \Delta,\ \frac{\Delta}{\lambda}) \qquad (7)$$

Based on 40 flume and field data, the following expression is proposed:

$$k_s = 3\ D_{90} + 1.1\ \Delta \left[1 - e^{-25\psi}\right] \qquad (8)$$

in which: $\psi = \Delta/\lambda$ = bed-form steepness

It may be noted that for $\psi = 0$ the plane bed value $k_s = 3\,D_{90}$ is obtained.
Figure 3 shows the equivalent form roughness as a function of bed-form dimensions.

Table 1. Comparison of prediction methods

758 flume data (score in error range ± 20%)	
1 Engelund-Hansen	65%
2 van Rijn	56%
3 White et al	54%

786 field data (score in error range ± 20%)	
1 van Rijn	74%
2 White et al	58%
3 Engelund-Hansen	47%

For field conditions the proposed method according to equations (4), (5) and (8) is superior to both other methods.

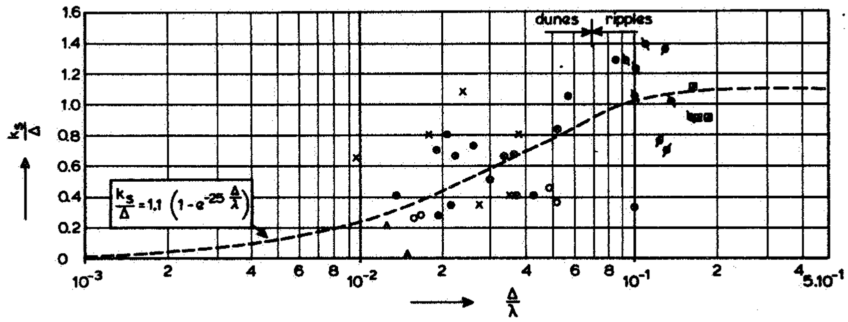

Fig. 3. Equivalent form roughness as a function of bed-form height and steepness

The Chézy-coefficient follows from:

$$C = 18\ \log\left(\frac{12\ R_b}{k_s}\right) \qquad (9)$$

in which: R_b = hydraulic radius of the bed.

5 VERIFICATION

The proposed functions were verified by comparing predicted and measured Chézy-coefficients using 1544 flume and field data. For comparison also the methods of Engelund-Hansen (1967) and White et al (1979) were applied to the selected data. In Table 1 the accuracy of the three methods is given in terms of the score (in %) of the predicted values in the error range ± 20% of the measured values.

REFERENCES

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