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APPLICATIONS OF SEDIMENT PICK-UP FUNCTION

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INTRODUCTION

In an earlier note (7) experimental work concerning the pick-up of sediment particles was reported and a new empirical pick-up function was proposed, which reads as:

$$E = 0.00033\ \rho_s (\Delta g D_{50})^{0.5} D_*^{0.3} T^{1.5} \quad (1)$$

in which E = pick-up rate in mass per unit area and time; ρ_s = sediment material density; $\Delta = (\rho_s - \rho)/\rho$ = relative density; ρ = fluid density; g = acceleration of gravity; D_{50} = particle size; $D_* = D_{50}(\Delta g/\nu^2)^{1/3}$ = dimensionless particle parameter; ν = kinematic viscosity coefficient; $T = (\tau_b' - \tau_{b,cr})/\tau_{b,cr}$ = dimensionless bed-shear parameter; τ_b' = effective bed-shear stress; and $\tau_{b,cr}$ = critical bed-shear stress according to Shields. In this note two applications of the proposed pick-up function are given.

APPLICATION 1: BED LOAD TRANSPORT

From simple kinematical considerations it can be shown that the bed-load transport, q_b, is given by (2):

$$q_b = E\lambda \quad (2)$$

Applying the "pick-up" definition according to Einstein (2), the λ-parameter represents the travel distance of a particle. Applying the definition of Yalin (13), which is adopted by the writer, the λ-parameter represents the jump or saltation length of a particle. In an earlier study (8), the writer proposed an expression for the jump or saltation length, λ, of bed-load particles, which reads:

$$\frac{\lambda}{D_{50}} = 3\ D_*^{0.6} T^{0.9} \quad (3)$$

Applying Eqs. 1, 2 and 3, the bed-load transport can be computed. When the stream bed is covered with bedforms the effective bed-load transport does occur at the upward sloping part of the bed forms. To be able to compute the effective bed-load transport, the effective bed-shear velocity at the upward sloping part must be estimated. In an earlier study (8), it has been shown that the effective bed-shear stress, τ_b', can be expressed by:

$$\tau_b' = \rho g \left(\frac{\bar{u}}{C'}\right)^2 \quad (4)$$

in which $C' = 18 \log (12\ R_b/3\ D_{90})$ = Chézy-coefficient related to the grains; $\bar{u}$ = depth-mean flow velocity; R_b = hydraulic radius of the bed according to the method of Vanoni-Brooks (11); and D_{90} = grain size corresponding to 90% finer.

To determine the predictive ability of Eq. 2, predicted and measured bed-load transport rates were compared. In all, 553 field and flume data were selected. Most of the data were selected from a compendium of solids transport data (6). As the reported data consist of total load data, only the experiments with a particle size larger than about 400 μm and a flow velocity smaller than 1 m/s were selected, assuming that for these conditions the sediment transport can be considered as merely bed-load transport. For nearly all data the ratio of the bed-shear velocity and the particle fall velocity was smaller than one ($u_*/w_s < 1$). In addition, some experiments from the Delft Hydraulics Laboratory (1) and Colorado State University (4) were used. For the latter data with particle sizes in the range 190 to 930 μm, the measured bed-load transport rates were obtained by reducing the measured total load rates with the measured suspended load transport rates.

For comparison with Eq. 2 two other typical bed-load formulas were applied to predict the bed-load transport rate, being the formulas of Meyer-Peter-Müller (5) and that of Frijlink (3).

The inaccuracy of the computed bed-load transport rate is given in terms of a discrepancy ratio, defined as:

$$r = \frac{q_{b,\text{computed}}}{q_{b,\text{measured}}} \qquad (5)$$

The percentage of r-values within the discrepancy ratio classes 0.75–1.5, 0.5–2.0 and 0.33–3.0 are presented in Table 1, which shows that the results of the three methods are about the same. For example, the percentage of r-values in the discrepancy ratio class 0.5–2.0 is about 60% for all three methods, which is a rather good result for a sediment transport theory (12). Regarding the proposed Eq. 2, the results are quite encouraging. Based on the present results, the equations for the sediment pick-up rate and the saltation length seem to predict values which have (at least) the right order of magnitude.

TABLE 1.—Measured and Computed Bed-Load Transport Rates

Source (1)	Number of tests (2)	Flow velocity (m/s) (3)	Flow depth (m) (4)	Median particle size (in 10^{-6} m) (5)	Temperature (°C) (6)	SCORES OF PREDICTED BED-LOAD TRANSPORT: $0.75 \le r \le 1.5$ van Rijn (7)	Meyer Peter Müller (8)	Frijlink (9)	$0.5 \le r \le 2$ van Rijn (10)	Meyer Peter Müller (11)	Frijlink (12)	$0.33 \le r \le 3$ van Rijn (13)	Meyer Peter Müller (14)	Frijlink (15)
Field														
Japanese channels	23	0.55–0.95	0.20–0.7	1,330–1,440	—	57	57	65	83	91	91	91	96	96
Mountain creek	43	0.50–0.80	0.10–0.40	900	15–25	23	74	70	44	95	95	83	100	100
Flume														
Guy et al.	39	0.3–1.0	0.15–0.30	190–930	8–34	33	51	46	49	87	82	67	100	95
Delft Hydraulics Laboratory	16	0.4–0.9	0.10–0.50	790	12–18	6	0	19	56	71	69	81	86	75
Stein	27	0.4–1.0	0.10–0.35	400	20–26	7	4	0	22	4	7	44	22	48
Meyer-Peter	17	0.45–0.9	0.10–0.20	1,000–1,500	—	6	6	6	12	41	6	30	76	88
Uswes-Sand	48	0.45–0.60	0.10–0.20	950	14–18	48	17	17	90	50	54	94	60	83
Uswes-Synthetic Sand	183	0.45–0.60	0.15–0.25	480–1,080	19–25	52	30	32	84	56	63	97	74	86
Singh	55	0.40–0.65	0.10–0.20	620	13–20	19	3	8	60	43	48	94	78	83
Znamenskaya	10	0.50–0.80	0.10–0.20	800	—	30	50	60	50	70	70	80	100	100
Southampton B	71	0.3–0.7	0.15–0.45	480	22–30	15	41	35	35	73	75	82	85	89
East Pakistan	21	0.45–0.70	0.15–0.30	470	25–30	33	14	5	67	33	38	81	67	57
Total	553					34%	31%	31%	63%	60%	62%	85%	81%	89%

APPLICATION 2: SUSPENDED LOAD TRANSPORT

A fundamental problem in the field of sediment transport is the modeling of the phenomena which govern the adjustment of sediment concentration profiles in a steady uniform flow which is initially free of sediment, as shown in Fig. 1. The adjustment of the concentration profiles can be described by a convection-diffusion type of equation.

For a steady and uniform two-dimensional (vertical plane) flow as shown in Fig. 1, the convection-diffusion equation can be described as:

$$\frac{\partial}{\partial x}(uc) - \frac{\partial}{\partial z}(w_s c) - \frac{\partial}{\partial x}\left(\epsilon_{s,x}\frac{\partial c}{\partial x}\right) - \frac{\partial}{\partial z}\left(\epsilon_{s,z}\frac{\partial c}{\partial z}\right) = 0 \quad (6)$$

in which c = sediment concentration; u = flow velocity; w_s = particle fall velocity; $\epsilon_{s,x}$ = longitudinal diffusivity; $\epsilon_{s,z}$ = vertical diffusivity; x = longitudinal coordinate; and z = vertical coordinate.

Assuming a scalar diffusion coefficient, a constant particle fall velocity and neglecting the longitudinal diffusive transport which is an order of magnitude smaller than the other terms (10), Eq. 6 can be simplified to:

$$u\frac{\partial c}{\partial x} - w_s\frac{\partial c}{\partial z} - \epsilon_s\frac{\partial^2 c}{\partial z^2} = 0 \quad (7)$$

The sediment (sand) diffusivity is described by the parabolic-constant distribution, that is characteristic for the logarithmic velocity profile, as follows (9):

$$\epsilon_{s,\max} = 0.25\,\beta\kappa u_* d, \quad \text{for } \frac{z}{d} \geq 0.5$$

$$\epsilon_s = \beta\kappa u_* z\left(1 - \frac{z}{d}\right), \quad \text{for } \frac{z}{d} < 0.5 \quad (8)$$

in which d = flow depth; β = ratio between diffusivity of sediment and fluid momentum; κ = Von Karman coefficient; and u_* = bed-shear velocity. The flow velocity is described by a logarithmic function (10). The boundary conditions are applied as follows:

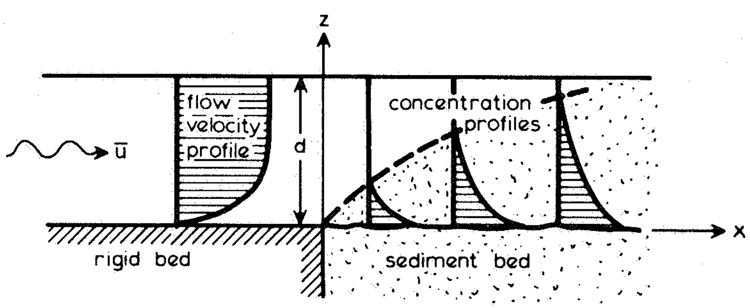

FIG. 1.—Sediment Pick-Up in Flow Without Initial Sediment Load

inflow: $c(0,z) = 0$ .... (9)

water surface: $\left(w_s c + \epsilon_s \frac{\partial c}{\partial z}\right)_{z=d} = 0$ .... (10)

bed surface: $\left(\epsilon_s \frac{\partial c}{\partial z}\right)_{z=2d} = E$ .... (11)

Eq. 11 specifies an upward sediment flux at level z = 2D (D = grain size) above the bed. The upward flux is assumed to be equal to the pick-up rate according to Eq. 1. As the flow is uniform and the sediment size is assumed to be constant, Eq. 1 specifies a constant pick-up rate and hence a constant upward sediment flux along the bed profile.

To solve Eq. 7, a finite element method based on weighted residuals according to the Galerkin method has been used (10). To model the relatively large concentration gradients near the bed, a grid refinement close to the bed has been used.

The mathematical results were compared with the results of a laboratory experiment, as shown in Fig. 1. The experiment was performed in a small flume with a length of 30 m, a width of 0.5 m and a depth of 0.7 m. The discharge was measured by a circular weir. The mean flow depth was 0.25 m and the mean flow velocity was 0.67 m/s. The bed material had a D_{50} = 230 μm and a D_{90} = 320 μm. The characteristic size of the particles in suspension was found to be about 200 μm (9), resulting in a characteristic fall velocity of 0.022 m/s (water temperature 9° C). The stream bed was covered with bed forms having a length of about 0.1 m and a height of about 0.015 m. Small Pitot tubes were used to determine the vertical distribution of the flow velocity, showing a logarithmic velocity profile. Water samples were collected simultaneously

TABLE 2.—Flow and Sediment Data of Flume Experiment

Variable (1)	Symbol (2)	Value (3)
Mean flow depth	d (m)	0.25
Depth-mean flow velocity	$\bar{u}$ (m/s)	0.67
	D_{50} (μm)	230
Particle diameter bed material	D_{90} (μm)	320
Particle fall velocity suspended material	w_s (m/s)	0.022
Water temperature	Te (°C)	9
Overall bed-shear velocity	u_* (m/s)	0.0477
Effective bed-shear stress	τ'_b (N/m^2)	1.2
Maximum diffusion coefficient (β = 1)	$\epsilon_{s,\max}$ (m^2/s)	0.0012
Equivalent roughness of Nikuradse	k_s (m)	0.01
Sediment density	ρ_s (kg/m^3)	2650
Fluid density	ρ (kg/m^3)	1000
Sediment pick-up rate	E (kg/sm^2)	1.05
Longitudinal grid size	Δx (m)	0.1
Vertical grid size	Δz (m)	variable (20 points)

at four locations by means of a siphon method to determine the spatial distribution of the sand concentrations. At each location five samples were collected at a height of about 0.015, 0.025, 0.05, 0.10 and 0.22 m above the average bed level. The most important flow, sediment and numerical parameters are reported in Table 2. The computed sediment pick-up rate is based on an effective bed-shear stress according to Eq. 4. Since the flow is uniform in longitudinal direction, the computed pick-up rate is constant in that direction.

Fig. 2 shows measured and computed sand concentrations at various heights above the bed as a function of longitudinal distance. The agreement is surprisingly good, particularly at a height of 0.015 m above the bed. The computed results show a more rapid adjustment of the near-bed sand concentrations compared with the measured concentrations. These deviations may be caused by the generation of a small scour hole with a maximum depth of about 0.1*d* just beyond the section with the

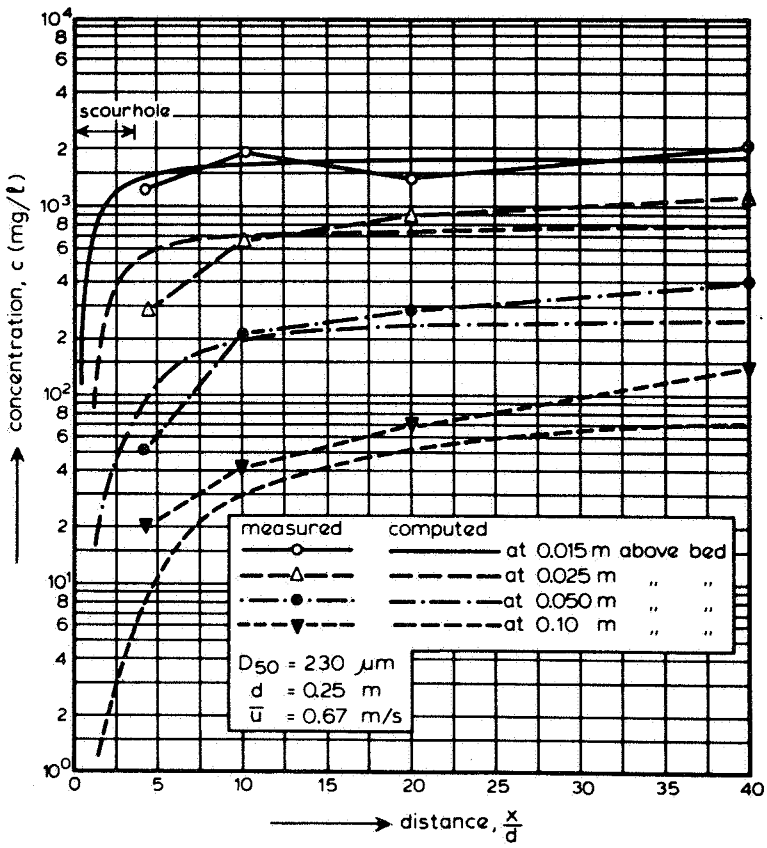

FIG. 2.—Longitudinal Distribution of Sand Concentrations

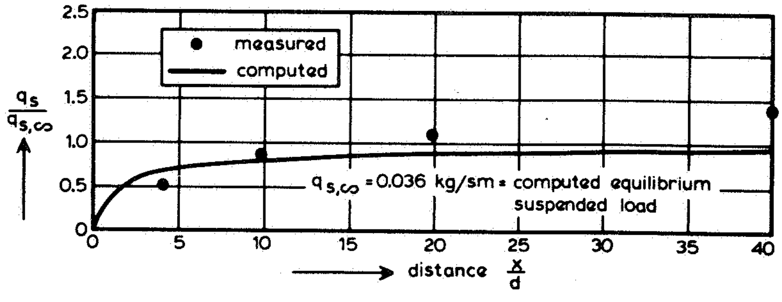

FIG. 3.—Longitudinal Distribution of Suspended Load Transport

rigid bed (see also Fig. 2). Consequently, the near-bed sand concentrations measured just downstream of the scour hole may be reduced somewhat. Another consequence of the scour hole is additional turbulence in the deceleration zone of the scour hole resulting in a larger adjustment length of the concentration profiles and the suspended load transport. This latter effect is clearly seen in Fig. 3 showing the measured and computed (depth-integrated) suspended load transport. As can be observed, the measured values are still increasing near the end of the flume, while the mathematical values are reaching equilibrium.

The results are quite encouraging, considering that no calibration has been applied. It seems that a constant upward sediment flux, which is independent of the local suspended load, can be applied successfully as a bed-boundary condition to compute the spatial distribution of the sand concentrations, resulting in a (stable) equilibrium value of the suspended load. Apparently, the upward sediment flux can be represented by the sediment pick-up rate expressed by Eq. 1. Definite conclusions, however, cannot be drawn because other physical phenomena, such as hindered settling of the suspended particles and damping of the turbulence (and hence the sediment diffusivity) which occur close to the bed, have not been modeled in the present approach. The overall influence may be small because the hindered settling effect yields larger concentrating while the turbulence damping effect leads to smaller concentrations (9).

Conclusions

By defining the bed-load transport as the product of the pick-up rate (Eq. 1) and the saltation or jump length (Eq. 3), the bed-load transport has been computed for 553 flume and field data, resulting in a score of 60% of the predicted values in the range 0.5–2 times the measured values; the bed-load formulas of Meyer-Peter-Müller and Frijlink produce similar results.

Applying a numerical solution of the convection-diffusion equation and applying the proposed sediment pick-up function (Eq. 1) as a bed-boundary condition, the adjustment of sand concentrations in an uniform flow without initial sediment load has been computed and compared with experimental results showing reasonably good agreement.

Appendix.—References

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