Quasi-3D and Fully-3D modelling of suspended sediment transport

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Introduction

The influence of different velocity and eddy viscosity fields on the computation of suspended sediment concentration and sediment transport rates, is presented here. Two different mathematical models which generate the velocity field were used: the WAQUA model which generates a steady 2-D horizontal velocity field based on the depth-integrated Reynolds equations, and the PHOENICS model that generates a steady fully 3-D velocity and eddy viscosity fields by solving the 3-D Reynolds equations simultaneously with the energy transport and energy dissipation rate equations (K-Epsilon approach). The velocity and eddy viscosity fields generated by these two models were used by the SUSTRA-3D model to obtain the suspended sediment concentration by solving the 3-D convection-diffusion equation. A straight channel with a spur dike at mid-length was chosen as the test case. Velocity, eddy viscosity and sediment concentration profiles, as well as sediment transport rates along two computational lines are presented. The final section presents the conclusions and recommendations for future work.

Velocity field models

The WAQUA model. This model generates a 2-D horizontal velocity field by solving the steady-state depth-averaged Reynolds equations. These equations are (using Einstein's summation notation):

\[
\frac{\partial}{\partial x_i} (\rho \overline{u}_i) = 0 \quad (2.1)
\]

Momentum balance:

\[
\frac{\partial}{\partial x_i} (\rho \overline{u}_i \overline{u}_j) + \frac{\partial}{\partial x_i} (\rho g h) + \frac{\partial}{\partial x_i} \left( h + Z_b \right) + \tau_{b,x_i} - T_{ij} + F_i = 0 \quad (2.2)
\]

where: \( x_i \) = space coordinates, \( (i,j = 1,2) \), \( \overline{u}_i \) = depth-averaged velocity in \( i \)-direction, \( F_i \) = Coriolis force component in \( i \)-direction, \( \tau_{b,x_i} \) = bed-shear stress in \( i \)-direction, \( T_{ij} \) = Depth-averaged stress tensor which includes

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viscous and turbulent stresses, $T_{12} = T_{21} = 0$, $T_{11} = \tau_{x} v^{2}(0)$, $T_{22} = \tau_{y} v^{2}(0)$, $\rho$ = water density, $h$ = water depth, $Zb$ = distance from the reference plane to bottom

The flow boundary conditions are as follows:
- The velocity profile at the entrance is prescribed.
- The normal derivative of the velocity is equal to zero at the outflow boundary section.
- The slip boundary condition is used at the solid boundaries (side walls and bottom).

Numerical solution of Eqs. (2.1) and (2.2), under the boundary conditions mentioned above, is obtained by using a finite-difference technique on a staggered grid.

The PHOENICS model. This model generates a 3-D velocity field and eddy viscosity coefficient field by solving the 3-D Reynolds equations simultaneously with the energy transport and energy dissipation rate equations. The complete set of equations, using the coordinate system shown in Fig. 1 and Einstein's summation notation is:

\[
\text{Mass balance:} \quad \frac{\partial U_{i}}{\partial x_{i}} = 0 \quad (2.3)
\]

\[
\text{Momentum balance:} \quad U_{i} \frac{\partial U_{j}}{\partial x_{i}} + \frac{1}{\rho} \frac{\partial (h + Zb)}{\partial x_{i}} + \frac{1}{\rho} \frac{\partial}{\partial x_{j}} \left( \tau_{ij} \right) = 0 \quad (2.4)
\]

where the Reynolds stresses can be written as:

\[
\frac{1}{\rho} \tau_{ij} = \nu \left( \frac{\partial U_{i}}{\partial x_{j}} + \frac{\partial U_{j}}{\partial x_{i}} \right) - u_{i} u_{j} \quad (2.5)
\]

\[
-u_{i} u_{j} = \tau_{ij} = \frac{3 \nu}{\rho} \left( \frac{\partial U_{i}}{\partial x_{j}} + \frac{\partial U_{j}}{\partial x_{i}} \right) - \frac{2}{3} K \delta_{ij} \quad (2.6)
\]

where: $\tau_{ij}$ = eddy viscosity or fluid mixing coefficient assumed to be a scalar quantity, $K$ = kinetic energy of turbulence per unit mass, $\delta_{ij}$ = Kronecker delta function

Energy transport equation read as:

\[
\frac{\partial (U_{i} K)}{\partial x_{i}} - \frac{\tau_{ij}}{\rho} \frac{\partial U_{j}}{\partial x_{i}} - \frac{\partial}{\partial x_{i}} \left( \frac{\partial K}{\partial x_{i}} \right) + E = 0 \quad (2.7)
\]

Energy dissipation rate transport equation read as:

\[
\frac{\partial (E)}{\partial x_{i}} = C_{1} (E) \frac{\tau_{ij}}{\rho} \frac{\partial U_{j}}{\partial x_{i}} - \frac{\partial}{\partial x_{i}} \left( \frac{\tau_{ij}}{\rho E} - \frac{\partial E}{\partial x_{i}} \right) + C_{2} E \frac{E}{K} = 0 \quad (2.8)
\]

\[
\tau_{ij} = C_{1} \frac{E}{\rho} \frac{E}{K} \quad (2.9)
\]
The following boundary conditions are prescribed:

- Inlet: \( u_1 = g_1(z); \frac{U_m}{h} = (1 - \frac{Z}{h}); \frac{U_m^3}{kz} \) (2.10)

- Bottom: \( \frac{\partial u}{\partial n} = 0; \frac{U_m^2}{C_u}; \frac{U_m^3}{kz} \) (2.11)

- Side walls: \( \frac{\partial u}{\partial n} = 0; \frac{\partial K}{\partial n} = \frac{\partial E}{\partial n} = 0 \) (2.12)

- Free surface: \( \frac{\partial u}{\partial z} = 0; \frac{\partial K}{\partial z} = \frac{\partial E}{\partial z} = 0 \) (2.13)

where: \( \kappa = \text{von Karman's constant} = 0.435 \), \( U_m = \text{bed shear velocity} = (\tau_b/\rho)^{0.5} \), \( \tau_b = \rho g u^2/C^2 \), \( C = \text{Chezy's coefficient}, \epsilon_u = 0.09 \)

Suspended sediment transport model (SUSTRA-3D)

Equation and boundary conditions. After the velocity field is obtained either by WAQUA or by PHOENICS, the suspended sediment concentrations are calculated by solving, numerically, the fully three-dimensional convection-diffusion equation. For steady-state conditions this equation is as follows:

\[
\frac{\partial}{\partial x_1} (u_1 c) - \frac{\partial}{\partial z} (w_s c) - \frac{\partial}{\partial x_1} (c_s \frac{\partial c}{\partial x_1}) = 0
\] (3.1)

where: \( c = \text{suspended sediment concentration}, u_1 = \text{velocity field}, w_s = \text{fall velocity of suspended sediment particles}, c_{s,i} = \text{sediment mixing coefficient} \)

The following set of boundary conditions are applied to Eq. (3.1):

- Inflow boundary: Along this boundary the concentration profiles are prescribed as the equilibrium concentration profile (L. van Rijn and Meijer, 1986b).
- Outflow and solid wall boundaries: The normal derivative of the concentration is equal to zero.
- Water surface: The net vertical sediment transport is equal to zero:

\[
[w_s c + c_{s,z} \frac{\partial c}{\partial z}]_{2-h} = 0
\] (3.2)

- Bottom boundary: Sediment concentrations at level \( z = a \) are prescribed, following van Rijn's approach (van Rijn 1984b, 1985, 1986):

\[
C_{a,e} = 0.015 \frac{d_{50}}{a} \frac{T^{0.5}}{D^0.3}
\] (3.3)

Mixing coefficient. When the WAQUA model is used, the horizontal mixing coefficients \( \epsilon_{iX} \) and \( \epsilon_{iY} \) were assumed to be constant and equal to 0.5 m²/s (see van Rijn and Meijer, 1986 for details). The vertical mixing coefficient was assumed to follow a parabolic-constant distribution (see van Rijn and Meijer, 1986):
\[\epsilon_{f,z} = \epsilon_{f,\text{max}} \left[ 1 - \left( \frac{1-2z}{h} \right)^2 \right] \quad \text{for} \quad z/h < 0.5 \]
\[\epsilon_{f,z} = 0.25 \beta \kappa U_{*} h \quad \text{for} \quad z/h \geq 0.5 \]  

(3.4)

When PHOENICS was used, the mixing coefficients were obtained through the K-Epsilon approach.

**Sediment transport.** Once the concentration profiles are obtained after solving Eq. (3.1), the sediment suspendent transport can be evaluated by

Suspended load: \( S_{s, x_1} = \int_a^h \left( u_{i, c} - \epsilon_{s, x_1} \frac{\partial c}{\partial x_1} \right) dz \)

(3.5)

**Logarithmic velocity profile.** Solution of Eq. (3.1) requires a fully 3-D velocity field, so when only a depth integrated velocity field (coming for WAQUA model in this case), is available, a logarithmic law is used to generate the 3D-velocity field. SUSTRA-3D generates a logarithmic profile according to:

\[u_i = \frac{U_i}{(Z_0/h) - 1 + \ln(h/Z_0)} \ln(z/Z_0)\]

(3.6)

where: \( U_i \) = depth-integrated velocity component, \( Z_0 \) = zero-velocity level (= 0.033 \( k_s \)), \( k_s \) = effective bed-roughness height, \( h \) = water depth

**Test case and results**

A straight channel, with a spur dike at the mid-length, 3000 m long, 1000 m wide, 6 m deep, carrying 4000 m³/s, was chosen as the test case. Fig. 1 shows a top view of the computational domain (curvilinear grid). In the vertical direction, the grid points were distributed according to an exponential expression

![Fig. 1, grid and computational lines 5 and 18.](image)

The following general input data were used for all the SUSTRA-3D runs: \( \rho_w \) = density of water = 1000 kg/m³, \( \rho_s \) = density of sediment = 2650 kg/m³, \( \omega_s \) = particle fall velocity = 0.0125 m/s, \( D_{50} \) = median particle


size = 0.2 mm, D90 = 90% percental of particle size = 0.3 mm, k80 = bed roughness height = 0.25 m, \( \kappa \) = von Karman's constant = 0.4, \( v \) = kinematic viscosity of the fluid = 0.013 cm²/s, \( \rho \) = bed porosity = 0.4

Velocity and eddy viscosity fields from WAQUA and PHOENICS. As mentioned before, PHOENICS is a sophisticated model, developed by CHAM corporation in London, that solves the velocity and eddy viscosity fields using the K-Epsilon approach (see Eqs. (2.3) to (2.13)). A calibration procedure was used to simulate the bed friction and to guarantee that the boundary conditions related to \( K \) and \( E \) are equivalent to those specified by Eqs. (2.10) and (2.11). Laminar and turbulent viscosity parameters were calibrated according to PHOENICS statements for specifying boundary conditions to generate acceptable final velocity and eddy viscosity profiles. The velocity patterns generated by PHOENICS (Figs. 2 and 3) are in good agreement with those generated by WAQUA although the eddy downstream of the dike is smaller than the one obtained from WAQUA. The eddy viscosity field generated by PHOENICS is quite different from the analytical expression defined by Equation (2.10), even for points near the entrance. The difference is much larger (one order of magnitude) for points near the free surface. For points near the bottom, PHOENICS gives eddy viscosity values close to those obtained by Eq. (2.10), if the near-bed velocity at \( z = 0.5 \) m is used to evaluate the bed-shear velocity \( (U_s) \), assuming a logarithmic law. (These differences in the eddy viscosities have a large effect on the sediment concentration and sediment transport rate results obtained from SUSTRA-3D).

Fig. 2, velocity field from WAQUA Fig. 3, velocity field from PHOENICS

Sediment concentrations and transport rates based on PHOENICS and WAQUA

Sediment concentrations and sediment transport rates are obtained by SUSTRA-3D by solving the convection-diffusion equation for sediment concentration (Eq. (3.1)) with the boundary conditions given by Eqs. (3.2) to (3.3) and the velocity and eddy viscosity fields computed either by PHOENICS or by WAQUA.

Sediment transport rates along computational lines 5 and 18 (see Fig. 1) are shown in Figs. 4 and 5. The effect of the eddy viscosity on the sediment concentrations and sediment transport rates can be observed in these Figures. Along computational line 5 upstream of the dike, sediment concentrations and
sediment transport rates based on PHOENICS velocities have smaller values than those based on WAQUA. The small differences in the velocity field are not sufficient to cause the large differences observed in the sediment concentration and sediment transport rates (Fig. 4). The primary cause of these differences is that the eddy viscosity profile of PHOENICS gives relatively small ε₉-values. This is probably due to the bed-boundary condition used for K and E. More research is needed to evaluate the real effect. Along the same computational line but downstream of the dike (including the recirculating zone), two effects need to be taken into account: eddy viscosities and the eddy size. The fact that a smaller eddy is generated by PHOENICS than by WAQUA, makes the velocities in WAQUA (for this region) larger than those in PHOENICS and therefore the concentrations computed with the WAQUA velocities are larger than those computed with the PHOENICS velocities. The large difference in the sediment transport rate (Fig. 4) is consistent with the combined effect of the velocity and eddy viscosity field differences.

Along computational line 18 upstream of the dike, PHOENICS gives velocities slightly larger than those given by WAQUA and the difference in the eddy viscosity between the two models is now smaller than along the previous computational line, with even slightly larger values given by PHOENICS than by WAQUA, for points near the bottom. With these values for velocities and eddy viscosities, SUSTRA-3D gives larger concentrations for velocities and eddy viscosities from PHOENICS than for those from WAQUA, especially for points near the bottom. The large differences in the sediment transport rate shown along this computational line downstream of the dike can be explained, once again, by the combined effect of the eddy viscosity profile and the eddy size. For this region, the eddy generated by WAQUA shows small velocities (even back flow) and with smaller eddy viscosities, the sediment transport rates are smaller than those calculated by values taken from PHOENICS.

Fig. 4, sediment transport line 5  Fig. 5, sediment transport line 18
Conclusions and recommendations for future work

- The differences in the sediment concentrations and sediment transport rates found in this paper are caused by the differences in the eddy viscosity field and the differences in the flow pattern downstream of the dike. The quasi-3D approach (WAQUA + SUSTRA-3D) gives reasonable results. The fully-3D approach (PHOENICS + SUSTRA-3D) gives less satisfactory results.

- Probably, the built-in PHOENICS function, to determine the bed-shear velocity yields values which are too small because a smooth-wall approach is applied. This function should be replaced by a rough wall approach based on a logarithmic law.

- The eddy size downstream the dike computed by WAQUA is much larger than that computed by PHOENICS. This may be caused by the larger eddy viscosities ($\nu_x = \nu_y = 0.5 \text{ m}^2/\text{s}$) used by WAQUA. The grid size in the recirculating zone (downstream of the dike) may have been too large for the PHOENICS model so that the flow separation after the dike could not be simulated adequately.

REFERENCES


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