The State of the Art in Sediment Transport Modelling

L.C. van Rijn

Introduction

The physical understanding and mathematical modelling of the water and sediment motion in rivers, estuaries and coastal waters have made much progress in recent years by various research institutes. This has led to a number of more or less ready to use numerical model systems, but it has also raised many new research questions.

According to the number of space dimensions and spatial orientation, four classes of models are considered: (quasi-) three-dimensional (3D), two-dimensional vertical (2DV), two-dimensional horizontal (2DH) and one-dimensional models (1D).

Three dimensional models, 3D

Introduction. Until recently, most applications of suspended sediment models have been restricted to transport processes in the two-dimensional vertical plane, mainly for economic reasons (computer cost). For computations at wide horizontal scales (estuaries, coastal zones, seas), usually, another approach has been followed applying depth-averaged flow models in combination with a simple equilibrium sediment transport formula (Boer et al, 1984). Such an approach is only valid when the horizontal length scale of the adjustment process of the local suspended sediment transport to the local equilibrium transport is smaller than the maximum allowable horizontal grid size of the applied mathematical model. If this latter requirement is not satisfied, it is of essential importance to represent the horizontal and vertical adjustment of the sediment transport process in the model. Basically, two types of modelling can be used: (1) the depth-integrated approach as introduced by Galappatti and Vreugdenhil (1985) and (2) the three-dimensional approach as applied by Sheng and Butler (1982), O'Connor and Nicholsoon (1988), Miller (1983), Wang and Adeff (1986), Van Rijn and Meijer (1988) and others. The application of the depth-integrated approach is limited to situations where the difference between the local true suspended sediment transport and the local equilibrium transport is relatively small (Wang, 1989), otherwise the results are not accurate. An advantage of the depth-integrated approach is the relatively low computer cost compared with that of three-dimensional models which are becoming increasingly popular because of impressive advancement of computer technology (memory size and speed). A good example of these latter developments is the full

\(^1\) DELFT HYDRAULICS, P.O. Box 152, Ennemloord, The Netherlands.
unsteady three-dimensional model for fluid velocities and sediment concentrations of Wang and Adeff (1986).

Usually, the 3D-models are only applied to predict the initial rate of sedimentation and erosion in a given situation for reasons of limited computed power. The initial models provide good insight into the short-term effects of a proposed structure (new harbour, closure of a channel etc.), but they are of limited value for the long-term morphological evolution, at least without interpretation of the results. This interpretation should be based on experience in similar situations and basic knowledge of morphological processes.

Equations. The 3D-mass balance equation for the suspended sediment reads as:

\[
\frac{\partial c}{\partial t} + \frac{\partial (uc)}{\partial x} + \frac{\partial (vc)}{\partial y} + \frac{\partial ((w-u)c)}{\partial z} = \frac{\partial}{\partial x} \left( \frac{\partial}{\partial x} (\frac{\partial ce_x}{\partial x}) \right) - \frac{\partial}{\partial y} \left( \frac{\partial}{\partial y} (\frac{\partial c}{\partial y}) + \frac{\partial (\frac{\partial ce_y}{\partial y})}{\partial y} \right) - \frac{\partial}{\partial z} \left( \frac{\partial}{\partial z} (\frac{\partial c}{\partial z}) + \frac{\partial (\frac{\partial ce_z}{\partial z})}{\partial z} \right) = 0
\]  

(1)

where: \(c\) = sediment concentration, \(u, v, w\) = fluid velocity components in \(x, y, z\) directions, \(e_S\) = sediment mixing coefficient, \(w\) = particle fall velocity, \(t\) = time.

To operate a 3D-model, the flow velocities, wave heights and mixing coefficients must be known a priori. Figure 1 shows a schematic representation of the computation procedure.

**Flow field.** To represent the flow field, two approaches are generally applied: 1. full 3D-solution of the 3D-Reynolds' equations in case of a complicated geometry with flow separation (Toro et al, 1989; Chum, 1987; Wang and Adeff, 1986) and 2. quasi 3D-solution in case of a simple geometry (De Vriend, 1987; Van Rijn-Meijer, 1988; Toro et al 1989).
The most simple quasi-3D approach is the application of a depth-averaged flow model in combination with logarithmic velocity profiles. Thus:

$$\frac{\partial}{\partial t}(\rho u) + \frac{\partial}{\partial x}(\rho u^2) + \frac{\partial}{\partial y}(\rho uv) = 0$$  \hspace{1cm} (2)

$$\frac{\partial}{\partial t}(\rho v) + \frac{\partial}{\partial x}(\rho u^2) + \frac{\partial}{\partial y}(\rho uv) + \rho g \frac{\partial}{\partial x}(h + z_B) +$$

$$- \rho u \bar{v} - D \left[ \frac{\partial^2 \bar{u}}{\partial x^2} + \frac{\partial^2 \bar{u}}{\partial y^2} \right] + \tau_{b,xy} + I F_x = 0$$  \hspace{1cm} (3)

$$\frac{\partial}{\partial t}(\rho w) + \frac{\partial}{\partial x}(\rho u^2) + \frac{\partial}{\partial y}(\rho uv) + \rho g \frac{\partial}{\partial y}(h + z_B) +$$

$$+ \rho u \bar{w} - D \left[ \frac{\partial^2 \bar{w}}{\partial x^2} + \frac{\partial^2 \bar{w}}{\partial y^2} \right] + \tau_{b,y} + I F_y = 0$$  \hspace{1cm} (4)

$$u = \frac{z_0}{h} - 1 + \ln \left( \frac{h}{z_0} \right) \ln\left( \frac{z}{z_0} \right)$$  \hspace{1cm} (5)

$$v = \frac{z_0}{h} - 1 + \ln\left( \frac{h}{z_0} \right) \ln\left( \frac{z}{z_0} \right)$$  \hspace{1cm} (6)

$$w = w_h + \int_0^h \frac{\partial u}{\partial x} dz + \int_0^h \frac{\partial v}{\partial y} dz$$  \hspace{1cm} (7)

where: $\bar{u}, \bar{v}, \bar{w}$ = depth-averaged flow velocity components, $u, v, w =$ local flow velocity components in $x, y, z$ directions, $h =$ water depth, $f =$ Coriolis parameter, $D =$ diffusion coefficient, $\rho =$ fluid density, $\tau_{b,xy}, \tau_{b,y} =$ bed-shear stresses (Chézy-approach) $F_x, F_y =$ external forces (wave-induced, wind-induced etc.).

A more detailed quasi-3D modelling of the flow field is presented by De Vriend (1987). Starting from the full 3D-Reynolds equations (integrated over the short wave period) and applying a gradient-type turbulence closure (scalar mixing coefficient), vertical similarity hypotheses have been made for all dependent variables. In this way the primary flow in the direction of the depth-averaged velocity can be distinguished from the secondary flow. Wave-induced and wind-induced effects are included. For example, Fig. 2 shows current vectors at three elevations above the bottom ($z = h$) for a nearshore current system.

**Fig. 2** Velocity vectors at three elevations above the bed ($z = h$)

**Fluid mixing coefficients.** The mixing coefficient can be represented by a horizontal ($\varepsilon_{s,x}, \varepsilon_{s,y}$) and a vertical ($\varepsilon_{s,z}$) component. General-
ly, the horizontal components are represented as constant values in the range $c_{a,y} = c_{b,y} = D = 0.1 - 1 \text{ m}^2/\text{s}$ (Van Rijn, 1987). The vertical mixing coefficient $c_{b,z}$ in boundary layer flow can be best represented by a parabolic distribution. A parabolic-constant distribution (parabolic in lower half and constant in upper half of the depth) can also be used. The latter approach yields a somewhat better representation of the concentration profile in the water surface region (Van Rijn, 1987).

**Boundary conditions.** To specify the boundary conditions of the transport model, information of the bathymetry, water depths and sediment characteristics (size, fall velocity, density etc.) is required. The most fundamental boundary condition is the process that controls the exchange of sediment particles at the bed. Three options are available to describe the exchange process:

- The convective flux is assumed to be equal to the equilibrium flux and specified as a known function of local hydraulic and sediment parameters:
  $\omega_s c_{a,e} = F(t_b, d_{50}, w_s, \ldots)$

  For sandy environments Van Rijn (1984) has proposed the following expression:
  \[ c_{a,e} = 0.015 \frac{d_{50}^{1.5}}{a^{0.3}} \]
  where $T = (t_b - t_{b,cr})/t_{b,cr}$, $D_b = d_{50}/(\rho_s - \rho)g/\nu^2$. \[ (9) \]

- The diffusive flux is assumed to be equal to the equilibrium flux and specified as a known function:
  \[ \frac{\partial c_a}{\partial z} = f(t_b, d_{50}, w_s, \ldots) \]

  This method usually is applied for silty and muddy conditions.

- The net flux is prescribed:
  \[ (w_a - c_{a,e}) \frac{2c_a}{\partial z} - a\omega_s (c_a - c_{a,e}) \]

  where $c_{a,e}$ = equilibrium concentration at reference level $z = a$ and $c_a$ = actual concentration at $z = a$.

The other boundary conditions are no net sediment flux at the water surface and no horizontal diffusive transport in a direction normal to a solid boundary.

**Solution method.** To solve the convection-diffusion equation, finite-difference and finite-element methods have been used. Schoellhamer (1988) applied a Lagrangian solution method. Efficient and accurate results require a grid refinement in the bottom region where the concentration gradients are relatively large. The following table provides information of the influence of the number of vertical grid points (applying a logarithmic vertical scale) on the accuracy of the computed concentrations and transport rates (Van Rijn, 1987).

The maximum error does occur in the water surface region, where the vertical grid size is maximum. The errors in the depth-integrated transport rate are much smaller because most of the material is transported in the nearbed region where the errors in the concentration are
small. In a sandy environment the number of grid point should be about 10, while in a muddy environment about 5 grid points can be used.

<table>
<thead>
<tr>
<th>number of points</th>
<th>max. error in concentration</th>
<th>error in depth-integrated susp. sed. transport</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$u_d/u_b=0.06$</td>
<td>$u_d/u_b=0.3$</td>
</tr>
<tr>
<td>20</td>
<td>0.2%</td>
<td>5%</td>
</tr>
<tr>
<td>10</td>
<td>1%</td>
<td>20%</td>
</tr>
<tr>
<td>5</td>
<td>5%</td>
<td>100%</td>
</tr>
</tbody>
</table>

Bed levels. Bed level changes can be determined from the depth-integrated mass-balance equation, yielding:

\[
\frac{\partial b}{\partial t} + \frac{x}{2} \left( \frac{\partial (q_{t,x})}{\partial x} + \frac{y}{2} \left( \frac{\partial (q_{t,y})}{\partial y} \right) \right) = 0
\]

where $b$ = bed level above a fixed datum, $p$ = porosity factor, $q_{t,x} = \int (uc - \epsilon_{s,x} \frac{ac}{dx}) dz$ = total depth-integrated sediment transport in $x$-direction, $q_{t,y} = \int (uc - \epsilon_{s,y} \frac{ac}{dy}) dz$ = total depth-integrated sediment transport in $y$-direction.

Applications. Results of the 3D-models of O'Connor and Nicholson (1988) and of Van Rijn et al (1989) are presented. O'Connor and Nicholson (1988) applied their 3D-model to compute the concentration field in a laboratory channel, which was partially blocked by a pier against one side-wall. Figure 3 shows measured and computed concentrations for 28 s after the dump. The sediment material consisted of polystyrene spheres ($\mu_d = 0.0017$ m/s). The concentrations were measured by means of an optical method. As can be observed, there is an acceptable degree of agreement with respect to the sediment cloud and the peak concentrations.

Fig. 3 Computed and measured concentrations (g/l) for 28 s after dump (O'Connor - Nicholson, 1988)

Van Rijn et al (1989) applied their 3D-model to compute the transport rates and initial bed level changes in a tidal area with velocities in the range of 0.6 to 1.5 m/s and depths in the range of 5 to 15 m. The
bed material consisted of sand with $d_{50} = 250$ µm. The bed roughness was found to be 0.5 m. The velocity field was computed by applying a 2D-model. Logarithmic velocity profiles were assumed to obtain a quasi-3D velocity field. Measured and computed mean velocities and transport rates in 6 locations during maximum ebb flow are reported in the following table.

<table>
<thead>
<tr>
<th>Location</th>
<th>Velocity (m/s)</th>
<th>Transport (kg/sm)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>measured</td>
<td>computed</td>
</tr>
<tr>
<td>A</td>
<td>0.94</td>
<td>0.97</td>
</tr>
<tr>
<td>B</td>
<td>1.25</td>
<td>1.26</td>
</tr>
<tr>
<td>C</td>
<td>1.28</td>
<td>1.30</td>
</tr>
<tr>
<td>D</td>
<td>1.07</td>
<td>1.33</td>
</tr>
<tr>
<td>E</td>
<td>1.28</td>
<td>1.42</td>
</tr>
<tr>
<td>F</td>
<td>1.11</td>
<td>1.22</td>
</tr>
</tbody>
</table>

|          | measured       | computed         | ratio |
| A        | 0.15           | 0.11             | 1.35  |
| B        | 0.9            | 0.9              | 1     |
| C        | 1.5            | 1.3              | 1.2   |
| D        | 2.7            | 1.6              | 1.7   |
| E        | 1.7            | 1.9              | 0.9   |
| F        | 1.3            | 1.0              | 1.3   |

Figure 4 shows initial bed level changes during ebb flow. The models of O'Connor-Nicholson (1969) and Van Rijn et al. (1989) have also been applied to compute transport rates in muddy conditions.

**Fig. 4** Initial bed level changes (Van Rijn et al., 1989)

Two-dimensional vertical models, 2DV

Introduction. Early attempts of two-dimensional vertical mathematical modelling for non-uniform conditions have been presented by Kalinske (1940) and Robbins (1943). Later a more general mathematical approach
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was presented by O'Connor (1971). Smith and O'Connor (1977) presented a two-dimensional vertical model based on the laterally-integrated momentum and continuity equations for the fluid and sediment phases, and an "one-equation" turbulence closure model to represent the fluid shear stresses and diffusion coefficients. Celik and Rodi (1984, 1985, 1988) presented a similar model as that of Smith and O'Connor. However, the former applied a "two-equation" turbulence closure model (K-Epsilon model). Bechteler and Schrumpf (1984) presented a relatively simple two-dimensional model neglecting vertical convection and horizontal diffusion. The fluid field was represented by logarithmic velocity profiles. Markovsky et al. (1985) presented a 2DV-model for mud concentrations. In the Netherlands two-dimensional modelling of suspended sediment was initiated by Kerssens (1974). Kerssens et al. (1977 and 1979) presented a two-dimensional vertical model for gradually varying flows neglecting vertical convection and horizontal diffusion. Logarithmic velocity profiles were used to represent the fluid velocity field. The vertical sediment mixing coefficients were represented by a parabolic-constant distribution. A concentration-type boundary condition was applied at the bed, assuming instantaneous adjustment to equilibrium conditions close to the bed. Later, the flow width was introduced to extend the applicability of the model to gradually varying flows in transverse direction. A (PROFILE-) method based on the application of flexible profiles (shape functions) was introduced to obtain a better description of the velocity profiles and the sediment mixing coefficients (Van Rijn, 1985, 1986, 1987).

Equations. In case of a sandy environment the mass balance equation (1) of the sediment phase can be simplified to

\[
\frac{\partial}{\partial x} (\omega_c) + \frac{\partial}{\partial y} (\omega_c) + \frac{\partial}{\partial z} (\omega_{c,g}) - \frac{\partial}{\partial z} (\omega_{c,g}) = 0
\]  (12)

because the time dependent term (\omega/\partial t) and the horizontal diffusion terms (\omega_{c,g}/\partial x, \omega_{c,g}/\partial y) are small with respect to the other terms (Kerssens et al., 1979).

Assuming the variables to be approximately constant in transverse direction (y), Equation (12) can be integrated in this direction yielding a width-integrated equation which can be applied to model the sediment transport in gradually varying channels (or stream tubes), as follows:

\[
\frac{1}{b} \frac{\partial}{\partial x} (bc) + \frac{\partial}{\partial z} (w_{c,g})c - \frac{\partial}{\partial z} (\omega_{c,g}) = 0
\]  (13)

Equation (13) can also be used for tidal conditions by schematizing the tidal cycle to a number of quasi-steady flow periods.

Flow velocities. The flow velocity profiles can be determined from various mathematical models depending on the complexity of the bathymetry. For complicated flows with largely perturbed velocity profiles including flow separation, a refined mathematical approach is of essential importance. The most accurate description can be obtained by applying the 2DV Reynolds' equations in combination with a two-equation (K-Epsilon) turbulence closure. The K-E model consists of transport equations for the turbulence kinetic energy (k) and its dissipa-
tion rate (Epsilon). These types of models were successfully applied by Rodi (1980), Celik and Rodi (1984) and by Alfrink and Van Rijn (1983). For long-term computations the K-Epsilon model is not attractive because of excessive computer cost. Coles (1965) and later Van Rijn (1967) developed simple flexible profile models to model the velocity profiles. A fundamental drawback of the profile models is the need for experimental data to calibrate the coefficients. Coles (1965) showed that the velocity profiles in a non-uniform flow can be described by using a linear combination of a logarithmic profile representing the law of the wall and a perturbation profile representing the influence of pressure gradients. Figure 5 shows measured and computed velocity profiles for a trench in a laboratory channel. The results of the K-Epsilon model and the Profile-model show similar deviations compared with measured values.

![Graphs showing measured and computed velocity profiles](image)

**Fig. 5** Computed and measured velocity profiles in a 2DV laboratory experiment

**Fluid mixing coefficients.** The fluid (and sediment) mixing coefficient can be best derived by the K-Epsilon model in case of a complicated bathymetry (flow separation). For long-term computations this model is not yet attractive. Therefore, Van Rijn used the K-Epsilon model to calibrate the more simple profile model (Van Rijn, 1987). The vertical distribution (profile) was represented by a parabolic-constant distribution: parabolic in the lower half and constant in the upper half of the depth. The longitudinal variation of the mixing coefficient was represented by a first order differential equation applying two calibration coefficients. Figure 6 shows computed mixing coefficients in a trench.
Boundary conditions. The boundary conditions for 2D models are similar to those of 3D models, see Eqs. (8), (9) and (10). Numerical solution methods are also similar to those of 3D models.

Applications. Two-dimensional vertical models have been applied to predict transport rates, sedimentation and erosion in rivers, estuaries and coastal waters. The applications in rivers usually are related to sedimentation of sand material in pipeline, tunnel trenches and settling traps for irrigation channels. Applications in estuaries and coastal waters require stream tube modelling, which means that the dimensions of the stream tube should be known from field measurements or mathematical flow models. The 2D-models have been successfully applied for sedimentation predictions of pipelines, tunnel trenches and navigation channels. In case of sandy conditions a quasi-steady approach can be used, which means that the tidal cycle is schematized to a fixed number of quasi-steady flow periods. In case of muddy conditions this approach cannot be applied because the 2c/ut-term is relatively important.

Celik and Rodi (1985, 1988) applied a 2D-model to compute the longitudinal development of sand concentration profiles in a laboratory channel. The flow velocities are derived from the K-Epsilon model. Good agreement between measured and computed concentrations can be observed (Fig. 7). Their model was also used to compute the sediment concentrations in a settling basin (Fig. 8).

Van Rijn (1986, 1987) applied a 2D-model to compute the sedimentation and migration of a trench in a laboratory flume. Three trenches with side slopes of 1:3, 1:5 and 1:10 were considered (Fig. 9). Good agreement between computed and measured results can be observed. The model of Van Rijn (1986b) was also used for sedimentation predictions in combined currents and waves.
Fig. 7 Computed and measured concentration profiles (Celik and Rodi, 1985)

Fig. 8 Computed concentration distribution in a settling basin (Celik and Rodi, 1985)

Fig. 9 Computed and measured bed level profiles of a trench (Van Rijn, 1987)
Toro et al (1989) computed the tidal variation of mud concentrations in a flow crossing a navigation channel. The fall velocity was modeled as \( \text{w}_f = \alpha c^{1.3} \text{w}_{s0} \) with \( \text{w}_{s0} \) = clear water value (= 10^{-4} \text{ m/s}) and \( \alpha \) = coefficient. Figure 10 shows mud concentrations at three elevations above the bed.

![Graph showing tidal variation of mud concentrations](image)

Fig. 10 Tidal variation of mud concentrations

Markošsky et al (1985) computed the temporal distribution (tidal cycle) of mud concentrations in the Weser Estuary in Germany applying a constant fall velocity and a constant vertical mixing coefficient. Figure 11 shows concentrations at 5 different depths close to the upstream end of the estuary. The critical bed-shear stresses for erosion and sedimentation were varied to study the effects on the concentration distributions.

![Graph showing mud concentrations in the Weser Estuary](image)

Fig. 11 Mud concentrations in the Weser Estuary
(left: \( \tau_{ce} = 0.09 \text{ N/m}^2; \tau_{cs} = 0.09 \text{ N/m}^2; \)
(right: \( \tau_{ce} = 0.03 \text{ N/m}^2; \tau_{cs} = 0.03 \text{ N/m}^2; \)
Markošsky et al, 1985)
Two-dimensional horizontal models, 2DH

Introduction. Two-dimensional horizontal sediment transport and bed evolution models are based on the depth-integrated equations of motions in combination with a sediment transport formula or in combination with a depth-integrated sediment model. The basic elements of 2DH-models are similar to those given in Fig. 1. Most applications are still related to the prediction of the initial rate of sedimentation and erosion for reasons of computer power (Boer et al., 1984 and McManus et al., 1986).

Equations. The flow field is described by Eqs. (2), (3), (4). In case of relatively coarse sand (say $d_{50} > 300 \mu m$) which are transported close to the bed, the sediment transport can be described by a simple formula, as follows:

$$q_s = F(V, \gamma, h, H_2, T, d_{50}, \text{ etc.})$$  \hspace{1cm} (14)

In case of transport of relatively fine material ($d_{50} < 300 \mu m$) with an adjustment scale larger than the applied grid scale of the model it is important to represent the vertical distribution of the concentration profiles. An attractive depth-integrated approach has been proposed by Galappatti and Vreugdenhil (1985). This method is based on the convection-diffusion equation including boundary conditions at the bed and at the surface. It is assumed that the velocity and concentration profiles have a distribution similar to that of the equilibrium profiles, which restricts the application of the model to gradually varying flow conditions. For example, the complete first-order solution for an one-dimensional channel reads as:

$$c = a \bar{c} + \beta \frac{h}{\omega} \frac{\partial \bar{c}}{\partial t} + \gamma \frac{\bar{u} h}{\omega} \frac{\partial \bar{c}}{\partial x}$$  \hspace{1cm} (15)

where $\bar{c}$ = depth-averaged concentration, $h$ = water depth, $\omega$ = fall velocity, $\bar{u}$ = depth-averaged velocity, $\alpha$, $\beta$, $\gamma$ = coefficients representing the vertical distribution effects (profile functions). The coefficients (functions of the vertical coordinate) can be determined in advance applying equilibrium profiles for the velocity, mixing coefficients and concentrations. Wang (1989) generalized this method to two horizontal dimensions. The boundary conditions for the concentration model are similar to those expressed by Eqs. (8), (9), (10). For mud transport a relatively simple depth-averaged approach can be applied because the concentrations are nearly constant in vertical direction (Ariathurai and Krone, 1976; Cole and Miles, 1983; Teissonn and Fritsch, 1988), yielding:

$$\frac{\partial \bar{c}}{\partial t} + \frac{\partial \bar{c}}{\partial x} + \frac{\partial \bar{c}}{\partial y} = \frac{1}{\omega} \frac{\partial}{\partial x} (\\hline H D \frac{\partial \bar{c}}{\partial x}) - \frac{1}{\omega} \frac{\partial}{\partial y} (\\hline H D \frac{\partial \bar{c}}{\partial y}) - S = 0$$  \hspace{1cm} (16)

where $h$ = depth, $D$ = diffusion coefficient, $S$ = source-sink term. The source-sink term accounts for erosion or sedimentation. Generally, the expressions of Krone (1962) and Partheniades (1965) are applied. Proper simulation also requires modelling of the consolidation process (Teissonn and Fritsch, 1988).
Applications. An example is given of a morphological model for coastal waters where the interaction between the currents, waves, sediment transport and the bed evolution is taken into account. The sediment transport is represented by a formula type of approach. The model is presented by Andersen et al., 1988. The application considers a cooling water intake at the coast with a water depth of 4 m (see Fig. 12). The area is exposed to waves from 2 directions with a wave height $H_{\text{rms}} = 1$ m in both directions. The intake discharge is constant at 100 m$^3$/s. The bed material is sand with $d_{50} = 200$ μm.

Figure 12 shows the initial wave field, current field and bed levels after 18 and 32 months.

Struiksnma et al. (1988) have presented a 2DH-model to compute the bed evolution in a river bend. The sediment transport formula of Engelund and Hansen (1967) was used taking slope effects ($\xi_{\text{a}}$) and secondary flow effects (bed shear stress makes a small angle with the current vector) into account. Figure 13 shows measured and computed bed level profiles in a river bend (Rhine). A point bar is present in the beginning of the inner bank (right bank).

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**Fig. 12** Initial wave and current field, bathymetry after 18 and 32 months (Andersen et al., 1988)

Finally, an example of the depth-integrated model of Wang (1989) is given. Wang used his model to compute the sediment transport distribution in partially closed channel with steady flow. The geometry is
Fig. 13 Measured and computed bed levels in a river bend.
(De Vries et al., 1989)

given in Fig. 14. The water depth is 6 m, the approach velocity is about 0.65 m/s. The bed material is $d_{50} = 200 \mu m$. The bed roughness is $k_s = 0.25 m$. Figure 14 shows computed transport rates along streamline B. The Esmor-model represents the depth-integrated model of Wang. The outrench-2D and 3D models are those of Van Rijn (1986) and Van Rijn-Meyer (1986, 1988). Good agreement between the three models can be observed in the acceleration zone upstream of the dam; the agreement in the deceleration zone is less good.

Fig. 14 Computed sediment transport rates in a partially closed channel (Wang, 1989)
One-dimensional models

Rivers and estuaries. One-dimensional models are most frequently used to simulate the large-scale morphological changes in rivers and estuaries. In the latter case the system of tidal flood and ebb channels is modelled as a network system in which only vertical bed level changes are considered.

The basic set of equations (per unit width) reads as:

\[
\begin{align*}
\frac{\partial \bar{u}}{\partial t} + g \frac{\partial \bar{u}}{\partial x} + g \frac{\partial h}{\partial x} + g \frac{\partial b}{\partial x} + g C^2 h &= 0 \\
\frac{\partial h}{\partial t} + g \frac{\partial h}{\partial x} + h \frac{\partial \bar{u}}{\partial x} &= 0 \\
\frac{\partial b}{\partial t} + \frac{\partial h}{\partial x} &= 0 \\
q_t &= F(\bar{u}, h, d_{50}, C, \ldots) \\
C &= F(\bar{u}, h, d_{50}, \ldots)
\end{align*}
\]  

(17) (18) (19) (20) (21)

in which: \( \bar{u} \) = depth-averaged flow velocity, \( h \) = water depth, \( z_b \) = bed level above datum, \( C \) = Chézy-coefficient, \( q_t \) = sediment transport rate, \( t \) = time, \( x \) = longitudinal coordinate.

If necessary, the non-uniformity of the bed material applying various size-fractions can be taken into account (Thomas, 1982; Ribberink, 1988, Armanini-Di Silvio, 1988). Adjustment effects related to suspended sediment transport can be taken into account by applying the asymptotic approach of Galappatti and Vreugdenhil (1985). Figure 15 shows some results of this latter approach compared with measurements and a 3D-model.

Detailed information of one-dimensional river models with respect to numerical solution methods and practical applications is given by Cunge et al (1980) and by Jansen et al (1979). A state of the art review is given by De Vries et al (1989).

Coastal waters. One dimensional models are also used to simulate the cross-shore bed evolution in coastal waters. Applying these models, problems like beach and dune erosion and beach nourishment can be studied. The basic elements of cross-shore profile models are: 1. wave propagation model (including breaking and bottom friction), 2. wave velocity model (asymmetrical orbital velocities), 3. current velocity model (wave-induced velocities), 4. sediment transport model (bed load, suspended load, gravity effects) and 5. bed evolution model.

Examples of this type of models are the models of De Vries and Ballard (1988), Wam (1988), Hoelvink and Stive (1988) and Steefel (1987). Figure 16 shows the wave height variation and the bed level changes for a laboratory experiment (\( d_{50} = 150 \mu m, T = 1.8 \, s, H_{rms} = 0.17 \, m \)).
Fig. 15 Computed and measured bed levels of a trench in a laboratory channel

Fig. 16 Cross Shore bed profiles (Nairn, 1988)

Figure 17 shows the beach profile evolution in front of a dune revetment (generation of a large scour hole). These examples show that cross-shore profile models can be useful engineering and research tools.

Fig. 17 Beach profile in front of revetment (Steetzel, 1987)
Conclusions

The main conclusion arising from this review is that the three and two dimensional models are making substantial progress at the various institutes. Most models are focusing on the short-term predictions (initial bed level changes) because of limited computer facilities (budget). The results of these models are encouraging at least from a research point of view. Long term predictions will become within reach when supercomputers are more generally available. Fundamental problems which remain to be solved are the schematization of the boundary conditions, especially in case of combined current and wave conditions. Calibration and verification of mathematical models require a detailed set of synoptic data. Much effort should be put in field surveys to obtain these data.

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