THREE-DIMENSIONAL MODELING OF SAND AND MUD
TRANSPORT IN CURRENTS AND WAVES

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SUMMARY

A three-dimensional mathematical model (SUSTIM) for the computation of suspended sand and mud transport in combined currents and waves is presented. The SUSTIM-model computes the non-steady sediment concentrations by solving numerically the 3-dimensional mass-balance equation representing the horizontal, vertical convection and diffusion and the gravitational transport of sediment particles. The fall velocity may be a constant value or a concentration dependent formula. Results of test computations showing the influence of the mud constant, the bed-boundary condition and the fall velocity are presented. Finally, a project computation is presented.

INTRODUCTION


At present stage of computer power, the 3D-models are only applied to predict the sediment transport, the deposition and erosion rates for initial conditions. These types of "initial" models provide good insight into the short-term effects of a proposed structure (new harbour, groynes etc).
BASIC EQUATIONS

Sediment concentrations

The SUSTIM-model computes the distribution of the sediment concentrations in time and space by solving numerically the threedimensional time-dependent mass-balance equation representing the horizontal and vertical convection, diffusion and settling (by gravity) of the suspended sediment particles. The fluid flow velocities, the wave heights and the sediment mixing coefficients (eddy viscosity coefficients) must be known a priori either from field measurements, laboratory scale measurements or from mathematical flow computations.

The three-dimensional mass-balance equation for the suspended sediment reads as:

$$\frac{\partial c}{\partial t} + \frac{\partial}{\partial x} (uc) + \frac{\partial}{\partial y} (vc) + \frac{\partial}{\partial z} ((w-w_s)c) - \frac{\partial}{\partial x} (r_{s,x} \frac{\partial c}{\partial x}) - \frac{\partial}{\partial y} (r_{s,y} \frac{\partial c}{\partial y}) +$$

$$- \frac{\partial}{\partial z} (r_{s,z} \frac{\partial c}{\partial z}) = 0 \quad (1)$$

in which: $c =$ sediment concentration, $u =$ local fluid velocity in $x$-direction (longitudinal), $v =$ local fluid velocity in $y$-direction (lateral), $w =$ local fluid velocity in $z$-direction (vertical), $w_s =$ fall velocity of sediment particles, $r_{s,x} =$ sediment mixing coefficient in $x$-direction, $r_{s,y} =$ sediment mixing coefficient in $y$-direction, $r_{s,z} =$ sediment mixing coefficient in $z$-direction, $t =$ time.

The concentration $c$ represents a time-averaged value with a time scale larger than the wave period but much smaller than the tidal period (say about 5 minutes).

Fluid velocities

The main objective is to describe the vertical and horizontal transport processes of suspended solids in the case of boundary layer flows in combination with small wind-generated waves (no vertical fluid density stratification). For these types of flow the fluid velocities may be represented as simple as possible by applying a depth-averaged
mathematical flow model in combination with logarithmic velocity profiles to obtain a quasi three-dimensional velocity field. The influence of wind-generated surface waves on the velocity profiles is not taken into account.

This simple approach restricts the model to conditions with a very gradually varying bottom topography (slopes not steeper than 1:20). The depth-averaged equations for horizontal free-surface flow in shallow water, read as:

**Mass balance**

\[
\frac{\partial}{\partial t} (\rho h) + \frac{\partial}{\partial x} (\rho \bar{v} u) + \frac{\partial}{\partial y} (\rho \bar{v} v) = 0
\]  

(2)

**Momentum balance**

\[
\frac{\partial}{\partial t} (\rho \bar{v} u) + \frac{\partial}{\partial x} (\rho \bar{v} u^2) + \frac{\partial}{\partial y} (\rho \bar{v} u v) = -\rho g h \frac{\partial}{\partial x} (h + z_b) - \bar{v}_b + \rho h \bar{v} \bar{e}_y t, x, h \left( \frac{\partial^2 \bar{u}}{\partial x^2} + \frac{\partial^2 \bar{u}}{\partial y^2} \right) + \sum F_x
\]

(3)

\[
\frac{\partial}{\partial t} (\rho \bar{v} v) + \frac{\partial}{\partial x} (\rho \bar{v} u v) + \frac{\partial}{\partial y} (\rho \bar{v} v^2) = -\rho g h \frac{\partial}{\partial y} (h + z_b) - \bar{v}_b + \rho h \bar{v} \bar{e}_x t, y, h \left( \frac{\partial^2 \bar{v}}{\partial x^2} + \frac{\partial^2 \bar{v}}{\partial y^2} \right) + \sum F_y
\]

(4)

in which: \(\bar{u}, \bar{v}\) = depth-averaged velocities in \(x, y\) directions, \(h\) = water depth, \(\rho\) = fluid density, \(f\) = Coriolis parameter, \(F_x, F_y\) = external driving forces per unit area (wind-generated, wave-generated), \(t\) = time, \(x, y\) = horizontal coordinates, \(z_b\) = distance from reference plane to bottom, \(\bar{e}_x, \bar{e}_y\) = depth-averaged horizontal mixing coefficient.

The vertical distribution of the horizontal fluid velocities \((u, v)\) is represented, as follows (Van Rijn, 1990):

\[
u = \frac{\bar{u}}{(z_0/h) - l + \ln(h/z_0)} \frac{\ln(z/z_0)}
\]

(5)

\[
v = \frac{\bar{v}}{(z_0/h) - l + \ln(h/z_0)} \frac{\ln(z/z_0)}
\]

(6)

in which: \(z\) = vertical coordinate above bed, \(z_0\) = zero-velocity level
\( k_{s,c} = \text{effective bed-roughness height related to current.} \)

It should be noted that the influence of external forces (wind, waves) on the velocity profiles is not taken into account at this stage. The basic effects of a longitudinal salinity gradient on the velocity profile (vertical circulation) resulting in relatively large near-bed velocities during the flood tide and relatively small near-bed velocities during ebb tide can be (roughly) represented by using a different apparent bed roughness for the flood and ebb tides. These latter values should be obtained from measured velocity profiles.

The fluid velocity \( (w) \) in vertical direction at height \( z \) above the bed are computed from the local mass-balance equation:

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \tag{7}
\]

yielding:

\[
w_{z+z_b} = w_{z+b} + \int_{z+z_b}^{z+b+h} \frac{\partial u}{\partial x} \, dz + \int_{z+z_b}^{z+b+h} \frac{\partial v}{\partial y} \, dz \tag{8}
\]

The vertical velocity at the water surface follows from the kinematic boundary condition:

\[
\frac{\partial}{\partial t} (h+z_b) + u \frac{\partial (h+z_b)}{\partial x} + v \frac{\partial (h+z_b)}{\partial y} - w_{z+b} = 0 \tag{9}
\]

Equation (8), together with equations (5), (6) and (9), is solved numerically to compute the vertical velocities.

**Mixing coefficients**

The horizontal mixing related to the currents is assumed to be constant over the depth. Generally, the current-related mixing effect is relatively large near recirculation zones.

Typical values of the horizontal mixing coefficient for shallow waters are in the range of 0.1 to 1 m²/s. The exact value of \( r_f \) is not of
essential importance, because the effect of $\epsilon_f$ on the computed velocities in the free-shear layer is small for $\epsilon_f$ in the range 0.1 to 1 m$^2$/s. The influence of $\epsilon_f$ on the velocities in the main flow is even smaller. Although the exact $\epsilon_f$-value (in the range 0.1 to 1 m$^2$/s) is not so important for a good representation of the recirculation zone, a good estimate of the $\epsilon_f$-values is still important because it directly influences the magnitude of the lateral diffusive transport of sediment to the recirculation zone.

When a wave field is superimposed on a current, the overall vertical sediment mixing coefficient is represented by addition of the wave-related and current-related mixing coefficients, as follows:

\[
(\epsilon_{s,z,\text{cw}})^2 = (\epsilon_{s,z,c})^2 + (\epsilon_{s,z,w})^2
\]

in which $\epsilon_{s,z,\text{cw}}$ = sediment mixing coefficient for currents and waves, $\epsilon_{s,z,c}$ = current-related sediment mixing coefficient, $\epsilon_{s,z,w}$ = wave-related sediment mixing coefficient.

It is assumed that the wave-related mixing is not modified by the presence of a current. Based on the analysis of equilibrium concentration profiles generated by waves, the wave-related sediment mixing coefficient in vertical direction was found to be (van Rijn, 1989):

\[
\epsilon_{s,z,w} = \epsilon_{s,\text{w,bed}} \quad \text{for } z \leq \delta
\]

\[
\epsilon_{s,z,w} = \epsilon_{s,\text{w,max}} \quad \text{for } z \geq 0.5h
\]

\[
\epsilon_{s,z,w} = \epsilon_{s,\text{w,bed}} + \left(\epsilon_{s,\text{w,max}} - \epsilon_{s,\text{w,bed}}\right)\left(\frac{z - \delta}{0.5h - \delta}\right) \quad \text{for } \delta < z < 0.5h
\]

in which: $\epsilon_{s,\text{w,bed}}$ = wave-related sediment mixing coefficient close to the bed, $\epsilon_{s,\text{w,max}}$ = wave-related sediment mixing coefficient in the upper half of the depth, $\delta$ = thickness of near-bed mixing layer (≈ 3 ripple heights).

Measured concentration profiles were analyzed to relate the characteristic parameters of the sediment mixing coefficient distribution to general wave parameters, yielding (van Rijn, 1989):
\[ \varepsilon_{s,w,\text{bed}} = 0.004 \frac{D_A}{\delta} \hat{u}_{b,w} \] (12)

\[ \varepsilon_{s,w,\text{max}} = 0.025 \frac{h H_s}{T_s'} \] (13)

in which:

- \( D_A \) = \( \delta_50 \left[ \left( \rho_s - \rho \right) / \left( \rho g v^2 \right) \right]^{1/3} \), particle size parameter,
- \( \hat{u}_{b,w} \) = peak value of the orbital velocity near the bed (according to linear wave theory),
- \( H_s \) = significant wave height,
- \( T_s' \) = peak period of waves (relative to current).

For (quasi) equilibrium boundary layer flow without waves the sediment mixing coefficient can be described by a parabolic-constant distribution, as follows (van Rijn, 1989):

\[ \varepsilon_{s,z,c} = \varepsilon_{s,c,\text{max}} - \varepsilon_{s,c,\text{max}} \left( 1 - \frac{2z}{h} \right) \quad \text{for } z < 0.5 \, h \] (14a)

\[ \varepsilon_{s,z,c} = 0.25 \beta \times u_{*,c} \, h \quad \text{for } z \geq 0.5 \, h \] (14b)

**Fall velocity**

The fall velocity of the sediment particles is represented, as follows:

\[ w_s = \text{constant} \] (15a)

or

\[ w_s = \alpha w_{s,0} \, c^\beta \] (15b)

or

\[ w_s = w_{s,0} \left( 1-c \right)^\gamma \] (15c)

in which:

- \( w_s \) = fall velocity
- \( w_{s,0} \) = fall velocity of an individual particle in clear water
- \( c \) = sediment concentration (volume)
- \( \alpha, \beta, \gamma \) = coefficients

Equation (15a), (15b) or (15c) can be selected by the user of the model. Usually, Equation (15a) is used for sand particles. Equation (15b) can be used to represent the flocculation of mud particles resulting in an
increase of the fall velocity as a function of concentration.

Equation (15c) represents the hindered settling effect. The coefficients should be specified by the user.

**BOUNDARY CONDITIONS**

**Computational domain**

The following specifications are required:
- bathymetry above a datum: \( z_b = f(x, y) \)
- water depths: \( h = f(x, y) \)
- wave height: \( H_w = f(x, y) \)
- wave period: \( T_w = f(x, y) \)
- depth-averaged fluid velocities: \( u, v = f(x, y) \)
- depth-averaged fluid mixing coefficient: \( \varepsilon_f = f(x, y) \)
- general flow, wave and sediment parameters (bed roughness, bed material size, fall velocity etc.).

**Inflow boundary**

The equilibrium concentration profiles are specified: \( c(z) = c_e(z) \), computed from \( w_s c + \varepsilon_s \frac{dc}{dz} = 0 \) and using equations (10), (11), (12), (13), (14), (15), (16), (17) or (18) and (19) or (20).

**Outflow boundary, bank boundary, streamline boundary**

Two options are available: (1) the normal derivatives of the concentrations are zero, \( \frac{\partial c}{\partial n} = 0 \), or (2) the local equilibrium concentrations are specified, \( c(z) = c_e(z) \).

**Water surface boundary**

The net vertical sediment transport is taken zero, or:

\[
[w_s c + \varepsilon_s \frac{3c}{\partial z} \frac{\partial^2 c}{\partial z^2} + h = 0] \tag{16}
\]

**Sediment bed boundary**

The bed boundary is selected at a small height (a), also known as reference level, above the mean bed level. In that case the bed boundary
condition (concentration and/or concentration gradient) may be represented by its equilibrium value assuming that there is an almost instantaneous adjustment to equilibrium conditions close to the bed. A good estimate for the reference level (s) is half the bed form height. In the SUSTIN-model, the reference level can be specified as a constant value (s = constant) or as a constant percentage of the local water depth (s/h = constant). This latter option should be preferred when the flow depth varies over a large range from small values (< 1 m) near the coast to large values (> 10 m) at deep water.

Two options are available to prescribe the bed boundary condition at the reference level z = a:

1. the local equilibrium bed concentration:

   \[ c_{z=0} \rightarrow c_{a,e} = F \text{ (local hydraulic and sediment parameters)} \]

2. the local equilibrium upward diffusive sediment flux:

   \[ E_{z=0} = \left[ - c_{a} \frac{\partial z}{\partial z} \right]_{z=0} = F \text{ (local hydraulic, sediment parameters)} \]

The equilibrium bed concentration \( c_{a,e} \) and the upward sediment flux \( E_{a,e} \) are prescribed by known functions that relate both variables to local near-bed hydraulic and sediment parameters.

For bed material consisting of sand the following two functions are available (optional) (Van Rijn, 1989):

\[
c_{a,e} = 0.015 \left( \frac{d_{50}}{D_a} \right)^{1.5} \tag{17}
\]

in which: \( T = (\tilde{\tau}_b,cw - \tilde{\tau}_{b,cr}) / \tilde{\tau}_{b,cr} \) = bed-shear stress parameter, \( \tilde{\tau}_{b,cw} \) = effective bed-shear stress due to combined currents and waves, \( \tilde{\tau}_{b,cr} \) = critical bed-shear stress (Shields).

\[
E_{a,e} = 0.015 \left( \frac{d_{50}}{D_a} \right)^{1.3} \tag{18}
\]

in which:

\( w_s \) = fall velocity of suspended sediment.
For bed material consisting of clay and silt the following two functions are available (optional); (Van Rijn, 1989): 

\[ c_{a,e} = \frac{M}{w_{s,0}} T \]  

(19) 

\[ E_{a,e} = M T \]  

(20) 

in which:

- \( M \) = material constant
- \( T = \frac{(\tau_{b,cw} - \tau_{b,cr})/\bar{c}_{b,cr}}{\tau_{b,cw}} \) - bed-shear stress due to combined currents and waves,
- \( \tau_{b,cr} \) = critical bed shear stress for erosion,
- \( w_{s,0} \) = fall velocity in clear water.

**BED LEVEL CHANGES**

**Mass balance equation**

Based on the computed sediment concentration field and flow velocity field, the initial bed level changes are computed from the depth-integrated mass balance equation:

\[ (1-p) \frac{\partial}{\partial t} (h \bar{c}_2) + \frac{\partial}{\partial x} (h \bar{c}_2 \bar{s}_x) + \frac{\partial}{\partial y} (h \bar{c}_2 \bar{s}_y) = 0 \]  

(21)

in which:

- \( \bar{c}_2 \) = depth-averaged concentration,
- \( \bar{s}_x = \bar{s}_a + \bar{s}_b \) = depth-integrated sediment transport,
- \( \bar{s}_a \) = depth-integrated suspended load transport,
- \( \bar{s}_b \) = bed load transport.

For (quasi) steady conditions the storage term \( \frac{\partial (h \bar{c}_2)}{\partial t} \) can be neglected.

**Suspended load transport**

The depth-integrated suspended load transport in \( x \)- and \( y \)-direction are computed as:
\[ s_{a,x} = \int_a^h (uc - c_{s,x} \frac{8c}{8x}) \, dz \]  
\[ s_{x,y} = \int_a^h (vc - c_{s,y} \frac{8c}{8y}) \, dz \]  

\textbf{Bed load transport}

The bed load transport for combined currents and waves in sandy conditions is represented by a simple formula, which represents the stirring effect of the waves, as follows (Van Rijn, 1989):

\[ s_{b,x} = 0.1 \, d_{50} \, \frac{u^*_{*,x}}{D^*} \frac{1.5}{0.3} \]  
\[ s_{b,y} = 0.1 \, d_{50} \, \frac{u^*_{*,y}}{D^*} \frac{1.5}{0.3} \]  

in which: \( u^*_{*,x} = g^{0.5} \frac{G}{C'} \) = effective current-related bed-shear velocity, \( u^*_{*,y} = g^{0.5} \frac{G}{C'} \), \( C' \) = grain-related Chézy coefficient = 18 \( \log(12h/3d_{90}) \).

\textbf{NUMERICAL SOLUTION METHOD}

The three-dimensional mass-balance equation for the suspended sediment is solved numerically by the so-called "Finite Volume Method" on a curvi-linear horizontal grid. In vertical direction the grid size decreases towards the bed to model the large concentration gradients near the bed as accurately as possible. Velocities and mixing coefficients, defined in the concentration nodes of the curvilinear grid, are transformed to values on a rectangular grid. By interpolation the transformed velocities and mixing coefficients are obtained on a staggered grid, which is shifted one half of the node distance in each direction. On the transformed grid the mass-balance equation is discretized and expressed in a set of linear equations for the concentration values in the interior nodes of the cells (finite volumes). The operator for the convective terms is treated with a cell-Reynold's number dependent upwind scheme. The system is solved by
applying a point Jacobi-like iterative method. Boundary conditions involving derivatives of the concentrations are implemented iteratively. The horizontal depth-integrated suspended sediment transport rates are computed by integration from the concentration and velocity fields. In case of non-steady conditions the iteration loop is executed within each time step similar to the procedure for steady conditions. A complication is that the water level may vary in time so that the physical domain changes within a time step. As the computational domain remains constant, the transformation of the equations between the two domains yields two extra effects. The first effect is taken into account by modifying the fall velocity. The second effect, caused by the change of the physical volume elements is implemented in the spatial discretization scheme.

TEST COMPUTATIONS

Steady flow

Herein, sensitivity computations for mud transport are presented. Similar computations for sand transport have been presented elsewhere (Van Rijn and Meijer, 1986, 1988, 1989). Nine test computations were executed for the case of steady flow in a straight channel partially closed by a dike, as shown in Figure 1. A curvi-linear grid schematization was applied. Mud concentrations and transport rates along stream lines A and B will be presented, hereafter.

![Figure 1](image)

grid points of concentrations in middle of cells

Figure 1. Grid schematization, streamlines A, B and C
Figure 2 shows computed velocity vectors according to Equations 2, 3 and 4. A large eddy zone can be observed downstream of the dike. Figure 3 shows the depth-averaged velocity along streamline B.

**Figure 2, Flow field**

![Flow field diagram]

- $H = 10$ m/s
- $Q = 4000$ m$^3$/s
- $h_0 = 60$ m
- $w = 6.18$ m
- $\Delta t = 0.25$ m
- $t = 0.05$ m

**Figure 3, Depth-averaged flow velocity along streamline B**

The input parameters, the fall velocity formula and the type of bed boundary condition of the test computations are reported in Table 1.
<table>
<thead>
<tr>
<th>Run</th>
<th>Number of vertical grid points NZ</th>
<th>Mud constant</th>
<th>Fall velocity clear water $w_{s,0} \text{ (m/s)}$</th>
<th>Fall velocity formula</th>
<th>Bed boundary formula</th>
</tr>
</thead>
<tbody>
<tr>
<td>C01</td>
<td>10</td>
<td>4.10$^{-9}$</td>
<td>5.10$^{-5}$</td>
<td>$w_s = w_{s,0}(1-c)^{4}$</td>
<td>$E_a = MT 1.5$</td>
</tr>
<tr>
<td>C02</td>
<td>10</td>
<td>4.10$^{-9}$</td>
<td>5.10$^{-5}$</td>
<td>$w_s = w_{s,0}(1-c)^{4}$</td>
<td>$E_a = MT 1.5$</td>
</tr>
<tr>
<td>C03</td>
<td>6</td>
<td>4.10$^{-8}$</td>
<td>5.10$^{-5}$</td>
<td>$w_s = w_{s,0}(1-c)^{4}$</td>
<td>$E_a = MT 1.5$</td>
</tr>
<tr>
<td>C04</td>
<td>6</td>
<td>4.10$^{-7}$</td>
<td>5.10$^{-5}$</td>
<td>$w_s = w_{s,0}(1-c)^{4}$</td>
<td>$E_a = MT 1.5$</td>
</tr>
<tr>
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<td>6</td>
<td>4.10$^{-7}$</td>
<td>5.10$^{-5}$</td>
<td>$w_s = w_{s,0}(1-c)^{4}$</td>
<td>$E_a = MT 1.5$</td>
</tr>
<tr>
<td>C06</td>
<td>6</td>
<td>4.10$^{-7}$</td>
<td>10$^{-5}$</td>
<td>$w_s = w_{s,0}$</td>
<td>$E_a = MT 1.5$</td>
</tr>
<tr>
<td>C07</td>
<td>6</td>
<td>10$^{-8}$</td>
<td>5.10$^{-4}$</td>
<td>$w_s = 2.82 \times 10^{5} w_{s,0} c^{1.3}$</td>
<td>$E_a = MT 1.5$</td>
</tr>
<tr>
<td>C08</td>
<td>6</td>
<td>4.10$^{-8}$</td>
<td>10$^{-4}$</td>
<td>$w_s = 2.82 \times 10^{5} w_{s,0} c^{1.3}$</td>
<td>$E_a = MT 1.5$</td>
</tr>
<tr>
<td>C09</td>
<td>6</td>
<td>4.10$^{-8}$</td>
<td>10$^{-4}$</td>
<td>$w_s = 2.82 \times 10^{5} w_{s,0} c^{1.3}$</td>
<td>$E_a = MT 1.5$</td>
</tr>
</tbody>
</table>

Table 1. Parameters of test computations

The parameters which have not been changed, are:

$Q = 4000 \text{ m}^3/\text{s}$, $c_{s,x} = c_{s,y} = 0.5 \text{ m}^3/\text{s}$, $k = 0.4$, $\nu = 0.000001 \text{ m}^2/\text{s}$, $\rho = 1000 \text{ kg/m}^3$, $\rho_s = 2650 \text{ kg/m}^3$, $d_{90} = 30 \mu\text{m}$, $d_{90} = 100 \mu\text{m}$, $\tau_{b,cr} = 0.1 \text{ N/m}^2$, $k_s = 0.001 \text{ m}$, $\beta = \phi = 1$.

**Influence of vertical grid size**

The effect of the grid size on the sediment concentration is negligible small for a region covering the accelerating zone and a region upstream of the dike up to the entrance. Downstream of the accelerating zone the effect becomes gradually important; near the exit the maximum difference is about 40%, with the finer grid case (C02 run) giving the larger values. The sediment transport rate (TRS) is less sensitive to the vertical grid size. Downstream of the accelerating zone, TRS-values from the finer grid are higher than those from the coarse grid, with a maximum difference of about 40% at the exit.

**Influence of mud constant M**

The runs C03, C04, C05 and C06 show the influence of the mud constant M of the bed-boundary condition (Equations (19) or (20)) on the computed concentrations and transport rates. The largest M-value yields the largest concentrations. The results are shown in Figure 4. It can be concluded that the differences are basically proportional to the changes.
in the value of $M$. An increment in $M$ by a factor of 2500 (C03 and C06) produces sediment concentrations and transport rates that differ by a factor of 2700 approx. This effect can be visualized as a shift (up or down) of the concentration and transport rates without distortion. Calibration data are required to determine the proper mud constant. The C06 run shows surface concentrations which are somewhat closer to the bed concentrations (more uniform profile) due to the hindered settling effect. Larger concentrations yields a smaller fall velocity according to the applied fall velocity formula. This effect is only noticeable for concentrations larger than 10000 mg/l.

Figure 4. Influence of mud constant on sediment concentrations; streamline A
Figure 5, Influence of bed boundary condition and fall velocity on sediment concentrations; streamline B

Figure 6, Influence of bed boundary condition and fall velocity on sediment transport; streamline B
Influence of bed boundary conditions

Runs C08 and C09 show the influence of the type of bed boundary condition (along streamline B): a gradient type (C08) or a concentration type (C09). A specified bed concentration yields a much larger increase of the concentrations and transport rate in the acceleration zone upstream of the dike as shown in Figures 5 and 6. The run (C08) with the specified concentration gradient shows two peaks in the concentrations near the dike. Probably, this is the result of numerical instability. Earlier research has shown that a gradient type boundary condition may introduce oscillations in a zone with rapidly varying flow conditions (Van Rijn, 1985).

Influence of fall velocity

Comparison of the results of the runs C02 and C08 presents information of the influence of the fall velocity (Figures 5 and 6). The fall velocity along the trajectory from 1600 to 3000 m is about 0.5 mm/s in runs C02 and in the range of 1 to 8 mm/s in run C08. Thus, the fall velocity in run C08 is a factor 2 to 16 larger than in run C02. The transport rate in run C08 decreases from 16 to 5 kg/sm (from 1600 to 3000 m along streamline B, which is the deceleration zone resulting in deposition) giving a relative decrease of about 70%. The transport rate in C02 decreases from 0.55 to 0.36 kg/sm over the same distance giving a relative decrease of only 35% due to the presence of a smaller fall velocity. It is noted that only relative values can be compared because of different input conditions. More research is necessary to get more detailed information of the influence of the fall velocity (concentration-dependent).

Influence of non-erodable bed

In natural conditions a hard rigid bed (coral reef, glacial boulder clay, rock) may locally be present preventing the erosion of sediment particles. In that case Equations (17) to (20) can not be used and should be replaced by:

\[ w_s + \alpha_s \frac{dc}{dz} \bigg|_{z=a} = 0 \]  

(24)
Figure 7 shows the uni-directional flow over a non-erodable dam (from \( x = 120 \) to \( 240 \) m) with a height of 2 m. The upstream water depth is 8 m. The depth-averaged velocity is 1 m/s at \( x = 0 \) m. The applied fall velocity is \( 5.5 \times 10^{-4} \) m/s. The upper figure shows the transport rate after 30 iterations and after 500 iterations giving a steady numerical solution with a constant transport rate as expected (dam has a non-erodable bed). Similar computations for an irregular three-dimensional bathymetry were not yet successful due to numerical problems. Further research of rigid bottoms in non-erodable and deposition conditions is necessary.

\[ \text{Figure 7. Influence of non-erodable bed} \]

**Non-steady flow**

The test case considered is a two-dimensional (straight) channel with a length of 6500 m, a width of 1 m, a depth of \( h_o = 10 \) m and a deepened section of 10 m, as shown in Figure 8.

The discharge is given by:
\[ Q = \dot{Q} \sin(\omega t) \quad (25) \]

The tidal variation of the water surface with respect to the mean level is given by:

\[ H = \hat{H} \sin(\omega t + \psi) \quad (26) \]

The depth-averaged flow velocity in each profile is given by:

\[ \bar{u} = Q(h_o + H) \quad (27) \]

in which:

- \( \omega = 2\pi/T \) = angular frequency
- \( T \) = tidal period (- 12 hours)
- \( \dot{Q} \) = peak discharge = 10 m³/s
- \( \hat{H} \) = peak tidal elevation = 2 m
- \( \psi \) = phase difference = 30°

In transverse direction uniform conditions are assumed to exist, which has been modelled by applying 3 grid points in that direction and assuming \( v = 0 \) m/s (zero velocity in transverse direction) and \( c_{x,y} = 0 \) m²/s (no diffusion). The velocity variation over the depth is assumed to be logarithmic. The bed boundary condition according to Eq. (19) has been used:

\[ (H = 4.10^{-8}, v_{s,0} = 10^{-4} \text{ m/s}, c_{b,cr} = 0.1 \text{ N/m²}). \]

The fall velocity is represented by \( v_s = \alpha v_{s,0} \beta \) (with \( \alpha = 2.82 \times 10^{-5}, \beta = 1.3 \) and \( v_{s,0} = 10^{-4} \text{ m/s} \)).

The computations have been made applying a time step of \( \Delta t = 1200 \text{ sec} \) and \( \Delta t = 3600 \text{ sec} \). The number of iterations per time step is 30. The number of grid points in vertical direction is 6. Figures 9 and 10 show computed concentrations for \( \Delta t = 1200 \text{ s outside } (I = 5) \) and inside the trench \((I = 46)\). The tidal variations of the concentrations are quite well represented. Inside the trench the peak concentrations are much smaller due to the reduced velocities. The concentration at the surface inside the trench shows small oscillations due to numerical instability, probably as a result of the relatively large vertical grid size near the surface. Figure 11 shows the computed depth-integrated transport rates.
The computed results for a time-step of $\Delta t = 3600$ s give a less smooth variation with time because the time step is too large (not shown).

**Figure 8, Channel geometry**

**Figure 9, Sediment concentrations upstream of the trench**

$(1 - 5, \Delta t = 1200$ s)
Figure 10, Sediment concentrations in middle of trench
\( (I = 46, J = 2) \), \( \Delta t = 1200 \) s

Figure 11, Depth-integrated sediment transport rates upstream of and in middle of the trench, \( \Delta t = 1200 \) s
PROJECT COMPUTATIONS - WATERFORD ESTUARY

As a part of the proposed new port development at Belview, Waterford, Ireland, it is necessary to increase the depth of the navigation channel to -6.0 m C.D. This requires cutting of channels through the Checkpoint Upper and Lower Bars (present levels are -3.5 m C.D.). The area of interest is the shipping channel at Upper and Lower Bar near Checkpoint in the river Suir at the confluence with the river Barrow (see Figure 12).

One of the alternative layouts studied is the construction of short groynes (surface level at 2.5 m C.D.) at the south bank in combination with a training wall at the north bank to increase the flow velocities in the Upper and Lower Bar channels (dredged to -6 m C.D.) in order to reduce the deposition rates. The channel width is 100 m.

The tidal currents were computed by numerical solution of Eqs. (2), (3) and (4). The model covered an area of approx. 7 km² of the rivers Suir and Barrow. The presence of the groyne in combination with the training wall yields an increase of the velocities of about 15% at Lower Bar Channel.

The SUSTIM-model was used to compute the sediment transport rates of the silt material (30 µm). The input values were: fluid density = 1023 kg/m³, sediment density = 2650 kg/m³, critical bed-shear stress = 0.1 N/m², bed roughness = 0.03 m, reference level at bed = 0.03 m, time step = 30 min. The fall velocity formula was: \( w_s = \frac{1}{2} \cdot (\frac{w_{s,0} \cdot \rho_s}{\rho_f}) \cdot \frac{1}{\rho_s} \) in which \( w_{s,0} = 0.03 \text{ m/s and } \rho_s = \text{volume concentration giving values of } w_s = 0.068 \text{ m/s for } c = 0.05 \text{ kg/m}^3 \) and \( w_s = 0.68 \text{ m/s for } c = 0.5 \text{ kg/m}^3 \). The mud constant \( M \) was calibrated using measured deposition volumes of Upper and Lower Bar channels during the period November 1989 to May 1990 (\( M = 4.10^{-10} \)).

Figure 12 shows the computed sediment transport rates at maximum ebb flow during springtide conditions. The largest values occur at the location of the channels near the groyne. Circulation patterns can be observed between the groyne and downstream of the groyne (typical deposition areas).

Figure 13 shows sediment concentrations and fluid velocities at three elevations above the bottom, the depth-integrated transport rates and the bed level changes in station 2 (middle of Upper Bar Channel).
The silt concentrations vary from 0.5 kg/m³ just after maximum ebb flow to about 0.005 kg/m³ after turning of the horizontal tide. The concentration profiles are almost uniform in vertical direction. The transport curve shows the typical behaviour of relatively rapid erosion and mixing processes and relatively slow deposition processes.

Figure 14 shows the deposition patterns (isolines) with values varying from 0.001 to 0.01 m per springtide. Deposition can be observed between the groyne, downstream of the groyne and in the deep pits (upto -20 m C.D.).

Figure 15 shows erosion patterns near the head of the groyne (accelerating flow) and at both channels through the bars due to the increased velocities, especially at Lower Bar Channel.

Figure 12. Silt transport rates at maximum ebb flow; springtide
Figure 13, Silt concentrations, velocities, transport rates and bed level changes in Upper Bar Channel; Spring tide
Figure 14, Deposition patterns; springtide

Figure 15, Erosion patterns; springtide
CONCLUSIONS

A three-dimensional mathematical model for suspended sand and mud transport in unsteady flow with or without short surface waves is presented.
Test computations for mud transport were made to study the influence of the mud constant, the bed boundary conditions and the fall velocity.
The sediment concentrations and transport rates are almost linearly dependent on the mud constant. Calibration (dredging volumes or deposition volumes) are required to determine the proper mud-constant for a certain project area.
A specified bed concentration yields a much larger increase of the concentrations and transport rate in accelerating flow than a specified bed concentration gradient (upward flux).
The fall velocity strongly influences the deposition process.
The presence of a non-erodable bed layer in accelerating flow can be modelled by imposing an equilibrium bed-boundary condition (Eq. 24) yielding a constant transport rate.
Test and project computations for mud transport in tidal conditions show a realistic behaviour of the sediment concentrations and transport rates.
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