SAND TRANSPORT BY CURRENTS AND WAVES; GENERAL APPROXIMATION FORMULAE

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Abstract: Simple approximation formulae have been developed to represent (with an accuracy of about a factor of 2) the bed load and suspended load transport under combined current and wave conditions, as described by the detailed TRANSPOR2000 model. The parameter ranges are: fine to medium coarse sand with d_{50} between 0.1 and 0.5 mm; depths between 0.25 and 20 m; current velocities between 0 and 2 m/s; relative wave heights H_s/h between 0 and 0.5.

INTRODUCTION

Coastal management dealing with morphological problems in the nearshore zone relies increasingly on predictions made by computational numerical models of sand transport and morphology. The morphological consequences of human intervention in the coastal system generally need to be evaluated on large spatial scales (10 to 100 km) and on long time scales (decades), requiring the availability of long term morphological models, such as the model package DELFT-MOR.

The objectives of the present study are to develop relatively simple approximation formulae for deph-integrated sediment transport capacity based on the engineering TRANSPOR2000 (TR2000) sand transport model for currents and waves (Van Rijn, 2000).

To develop the approximation formulae, the following approach has been used:

- define the practical range of interest, being: fine to medium coarse sand with d_{50} between 0.1 and 0.5 mm; depths between 0.25 and 20 m; current velocities between 0 and 2 m/s; relative wave heights H_s/h between 0 and 0.5;
- use the TR2000 model to compute the bed load and suspended load transport for this practical range of conditions;
- develop a set of relatively simple formulae for the bed load transport vector and the suspended load transport vector, which can represent the values of the TR2000 model within a factor of 2.

The bed-load transport vector due to both current and wave effects (including wave asymmetry) will be decomposed in a current-related contribution ($q_{b,c}$ in the current direction) and a wave-related contribution ($q_{b,w}$ in the wave direction; following or opposing depending on conditions).

The suspended load transport represents the current-related contribution due to advective processes ($q_{s,c}$ in the current direction) and the wave-related contribution mainly due to wave asymmetry effects ($q_{s,w}$ in the wave direction; always onshore in present approach).

DEFINITIONS

Depth-integrated sand transport is herein defined to consist of:

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- bed load transport, which is the transport of sand particles in the wave boundary layer (thickness of about 0.01 m) in close contact with the bed surface;
- suspended load transport, which is the transport of sand particles above the bed load layer (of about 0.01 m).

The suspended load transport can be determined by depth-integration of the product of sand concentration and fluid velocity from the top of the bed load layer (at about 0.01 m above the bed) to the water surface.
Herein, the net (averaged over the wave period) total sediment transport is obtained as the sum of net the bed load (q_b) and net suspended load (q_s) transport rates, as follows:

$$q_{tot} = q_b + q_s \qquad (1)$$

For practical reasons the suspended transport will be subdivided in current-related and wave-related transport components. This division is necessary to study the effect of phase differences between velocity and suspended sediment. For bed load transport of particles larger than about 0.2 mm such an approach is not required, because there is an almost instantaneous response of bed load concentrations to near-bed velocity (Ribberink, 1998).

BED LOAD TRANSPORT

Basic formulations

The net bed-load transport rate in conditions with uniform bed material is obtained by time-averaging (over the wave period T) of the instantaneous transport rate using a bed-load transport formula (quasi-steady approach), as follows:

$$q_b = (1/T) \int q_{b,t}\, dt \qquad (2)$$

with $q_{b,t}$ = F(instantaneous hydrodynamic and sediment transport parameters).

The applied bed-load transport formula is a parameterization of a detailed grain saltation model representing the basic forces acting on a bed-load particle for steady flow (Van Rijn, 1984a, 1993). This approach has been generalized to the regime of combined current and wave conditions by using the concept of the instantaneous bed-shear stress. The instantaneous bed-load transport rate (kg/s/m) is related to the instantaneous bed-shear stress, which is based on the instantaneous velocity vector (including both wave-related and current-related components) defined at a small height above the bed. The formula applied in the TR2000 model, reads as:

$$q_b = \gamma\, \rho_s\, d_{50}\, D_*^{-0.3}\ [\, [\tau'_{b,cw}/\rho]^{0.5}\ [\tau'_{b,cw} - \tau_{b,cr})/\, \tau_{b,cr}\,]^{\eta} \qquad (3)$$

in which: $\tau'_{b,cw}$= instantaneous grain-related bed-shear stress due to both currents and waves = $0.5\, \rho\, f'_{cw}\, (U_{\delta,cw})^2$, $U_{\delta,cw}$= instantaneous velocity due to currents and waves at edge of wave boundary layer, f'_{cw} = grain friction coefficient due to currents and waves= $\alpha\beta f'_c + (1-\alpha) f'_w$, f'_c = current-related grain friction coefficient, f'_w = wave-related grain friction coefficient, α= coefficient related to relative strenght of wave and current motion, β= wave-current-interaction coefficient, $\tau_{b,cr}$= critical

bed-shear stress according to Shields, ρ_s= sediment density, ρ = fluid density, d_{50} = particle size, D_* = dimensionless particle size, γ= coefficient= 0.5, η= exponent= 1.
The grain roughness is assumed to be: $k_{s,grain}=\varepsilon d_{90}$ with ε=3 for d_{50}<0.5 mm; ε=1 for d_{50}>1 mm and ε=3 to 1 for intermediate values (Van Rijn, 1984c, 1993). The bed-load transport is assumed to be mainly affected by the grain roughness, but the overall bed-form roughness also has some (weak) influence on the bed-load transport in case of combined steady and oscillatory flow because of its effect on the near-bed velocity profile.

Equation (3) is based on the assumption that the sediment particles respond instantaneously (quasi-steady) to the oscillatory fluid motion near the bed. The net transport rate will always be in the direction of the largest peak orbital velocity. This assumption is reasonably valid for the sheet flow regime with sediment particles larger than about 0.2 mm (Ribberink and Al-Salem, 1994; Dohmen-Jannssen, 1999). Ribberink and Chen (1993) have found that the net transport rate is against the direction of the largest peak orbital velocity for fine sediment (0.13 mm) in the sheet flow regime. This deviation (unsteady behaviour) is caused by phase-lag effects; the sediment concentration will lag behind the fluid velocity if the response time of the sediment is not small compared to the oscillation period. The entrainment of particles from the bed into the flow and the settling of the particles from the flow to the bed will take time for conditions with fine sediment.
Proper predictive modelling of the wave-related (oscillating) transport components basically requires an accurate description of the near-bed orbital fluid velocity, especially in conditions with shoaling and breaking waves (non-linear wave motion). Herein, the modified Isobe-Horikawa method (Grasmeijer and Van Rijn, 1998) has been applied.

Input data
The input data of the TRANSPOR2000 model are:

h	= water depth (m)
v_R	= depth-averaged and time-averaged current velocity due to tide, wind, waves (m/s)
u_r	= depth-averaged and time-averaged return current below wave crest (m/s)
u_b	= time-averaged velocity at edge of wave boundary layer generated by waves (m/s)
H_s	= significant wave height (m)
T_p	= peak period of wave spectrum (s)
φ	= angle (anti-clockwise) between current direction and wave propagation direction (0 - 360 degrees; 0=360=following, current and wave in same direction; 180=opposing, wave direction opposite to current direction)
d_{50}	= median particle diameter of bed material (m)
d_{90}	= 90% particle diameter of bed material (m)
d_s	= representative particle diameter of suspended sediment (m)
P_{mud}	= percentage of mud in bed material (in %)
$k_{s,c}$	= current-related bed-form roughness height (m)
$k_{s,w}$	= wave-related bed-form roughness height (m)
S_a	= fluid salinity (fresh water = 0 promille; sea water = 30 promille)
B_{slx}	= tangens of bed slope in current direction
B_{sly}	= tangens of bed slope normal to current direction

The velocity components v_R and u_r are composed to one vector value, yielding:
$v= (v_R^2+u_r^2+2v_Ru_r\cos(\varphi))^{0.5}$, which is used to compute the current-related parameters.

Approximation method

Equation (3) requires numerical (intra-wave period) computation of the bed load transport. As an approximation method, the following expression (4) is given for the time-averaged (over the wave period) bed-load transport vector due to currents and waves:

$$q_b = 0.006\ \rho_s\ w_s\ d_{50}\ M^{0.5}\ \ M_e^{0.7} \qquad (4)$$

with:
q_b = bed-load transport (in kg/s/m)
$M=(v_{eff})^2/((s-1)g\ d_{50})$= sediment mobility number due to waves and current (-),
$M_e=(v_{eff}-v_{cr})^2/((s-1)g\ d_{50})$= excess sediment mobility number (-),
$v_{eff}=(v_R^2+U_{on}^2)^{0.5}$= effective velocity due to currents and waves,
$U_{on}=U_{\delta,f}$= near-bed peak orbital velocity (m/s) in onshore direction (in wave direction) based on significant wave height (Isobe-Horikawa method, Grasmeijer and Van Rijn, 1998),
v_R= magnitude of depth-averaged current velocity vector (m/s),
$v_{cr}= 0.19(d_{50})^{0.1}\log(4h/d_{90})$= critical velocity (m/s) for $0.0001<d_{50}<0.0005$ m,
$v_{cr}= 8.5\ (d_{50})^{0.6}\log(4h/d_{90})$= critical velocity (m/s) for $d_{50}\geq0.0005$ m,
d_{50}, d_{90}= particle size characteristics (in meters),
w_s= fall velocity of d_{50} of bed material (m/s),
$s=\rho_s/\rho_w$= relative density (-),

Equation (4) has been fitted to Equation (3) by adjusting the coefficients of the approximation formula for particle sizes between 0.25 and 0.4 mm, water depths between 0.25 and 20 m, wave heights up to 2.5 m, mean velocities up to 2 m/s (see Van Rijn, 2001).
Assumptions used are: $d_{90}=2d_{50}$, d_{50}= median diameter of bed material, $k_{s,w}=k_{s,c}$=bed roughness= 0.03 m, wave-current angle= 90^o, temperature=15 ^oC, salinity=30 promille. The approximation method yields values, which are generally within a factor 2 of those based on the numerical method. Equation (4) represents the magnitude of the bed-load transport vector. The direction of this vector depends on: the current direction, the wave direction and the relative strength of the current (v_R) and peak orbital velocity (U_{on}). In the present study only one wave-current angle (=90^o) has been used. Analysis of sensitivity computations shows that the magnitude of the bed-load transport vector is about twice as large for a wave-current angle of 0^o, and about 50% larger for a wave-current angle of 180^o compared to the bed-load transport value for a wave current angle of 90^o.

Equation (4) only represents the magnitude of the bed load transport vector. The transport components ($q_{b,c}$ and $q_{b,w}$) in the wave and in the current directions (assuming angle of 90^o) can be derived from:

$$q_b^2=q_{b,c}^2+q_{b,w}^2 \qquad (5)$$

with:
φ= angle between current and wave direction, q_t= bed load transport vector, $q_{b,c}$= bed load transport in current direction, $q_{b,w}$= bed load transport in wave direction, q_b = bed load transport vector.
The magnitudes of the bed load transport in the current direction and in the wave direction are assumed to be related by:

$$|q_{b,w}| = r\ |q_{b,c}| \qquad (6)$$

with: $r = (|U_{on}| - v_{cr})^3/(|v_R| - v_{cr})^3$, $q_{b,w} = 0$ if $r < 0.01$ and $q_{b,c} = 0$ if $r > 100$

Using Eqs. (5) and (6), it follows that: $q_{b,c} = q_b/(1+r^2)$ (7)

The $q_{b,c}$ and $q_{b,w}$ values are applied in the prevailing wave and current directions (also if angle $\neq 90^o$).

SUSPENDED LOAD TRANSPORT

Definitions

The net time-averaged depth-integrated suspended sand transport is defined as the sum of the net current-related ($q_{s,c}$) and the net wave-related ($q_{s,w}$) transport components, as follows:

$$q_s = q_{s,c} + q_{s,w} = \int vc\, dz + \int <(V-v)(C-c)> dz \qquad (8)$$

in which: $q_{s,c}$ = time-averaged current-related suspended sediment transport rate and $q_{s,w}$ = time-averaged wave-related suspended sediment transport rate (oscillating component), v= time-averaged velocity, V= instantaneous velocity vector, C= instantaneous concentration and c= time-averaged concentration and $< >$ averaging over time, $\int$ the integral from the top of bed-load layer to the water surface.

The current-related suspended transport ($q_{s,c}$) is defined as the advective transport of sediment particles by the time-averaged (mean) current velocities (longshore currents, rip currents, undertow currents). Thus, the transport of sediment which is carried by the steady flow. In conditions with waves superimposed on the current, both the current velocities and the sediment concentrations will be affected by the wave motion. It is known that the wave motion reduces the current velocities near the bed, but the near-bed concentrations are strongly enhanced due to the stirring action of the waves. These effects are included in the current-related transport.

The wave-related suspended sediment transport ($q_{s,w}$) is defined as the transport of sediment particles by the high-frequency oscillating fluid components (cross-shore orbital motion). Low-frequency transport contributions are herein neglected. The wave-related transport component is defined in the plane of orbital motion.

For practical reasons the current-related and the wave-related transport components are studied separately. Furthermore, this allows the evaluation of the relative magnitude of both components, which is of significant importance for modelling purposes.

Wave-related suspended transport

Basic formulations

Modelling of the wave-related suspended transport ($q_{s,w}$) for a sand bed covered with ripple-type bed forms, basically requires the simultaneous (numerical) solution of both the time-dependent momentum equation for the oscillatory fluid flow and the time-dependent advection-diffusion equation for suspended sediment particles.

For the two-dimensional vertical plane this latter equation reads as:

$$\partial C/\partial t + \partial[UC- (\varepsilon_{s,x}\partial C/\partial x]/\partial x + \partial[(W-w_s)C- (\varepsilon_{s,z}\, \partial C/\partial z]/\partial z= 0 \quad (9)$$

with: C= instantaneous sand concentration (volume); U, W= horizontal and vertical instantaneous fluid velocities; w_s= fall velocity of suspended sand; $\varepsilon_{s,x}$, $\varepsilon_{s,z}$ = sediment mixing coefficient in horizontal x and vertical z directions; t= time; x= horizontal coordinate and z= vertical coordinate. The oscillatory flow along a rippled bed is rather complicated due to the generation, advection and diffusion of the near-bed vortices including the sediment particles carried by the vortices. Numerical simulation of the detailed vortex motions requires the application of sophisticated turbulence-models on a fine grid structure. Furthermore, the shape and dimensions of the ripples should be known a priori (boundary conditions). Using this approach, the instantaneous fluid flow and suspended transport due to combined steady and oscillatory flow over a rippled bed can be solved in an integrated way, which is a great advantage of this method. A major limitation is the relatively large computational time involved, when it is applied in a numerical morphological model system with feed back to changing bed levels and hence hydrodynamics (loop system). This detailed approach is at an early stage of research and beyond the scope of the present study.

For a plane bed without bed forms the advection-diffusion equation for the suspended concentrations can be simplified to:

$$\partial C/\partial t + \partial[(-w_s)C- (\varepsilon_{s,z}\, \partial C/\partial z]/\partial z= 0 \quad (10)$$

This unsteady model approach has been applied by many researchers to simulate the suspended concentrations inside and outside the wave boundary layer over a plane bed (see overview of Dohmen-Janssen, 1999). This approach may also be used to simulate the time-averaged sand concentrations over a rippled bed, provided that the overall effect of the bed forms on the sediment mixing coefficients is taken into account (Chung and Grasmeijer, 1999; Grasmeijer, 2002). Their results also show that the wave-related suspended transport can not be simulated accurately by Equation (10).

Herein, an engineering approach is proposed to estimate the wave-related suspended transport. This method (implemented in the TR2000 model) has been introduced by Houwman and Ruessink (1996). Experimental data are required to determine the empirical coefficient involved.

The wave-related suspended transport component is modelled as:

$$q_{s,w} = \gamma\ [(U_{on})^4-(U_{off})^4]/[(U_{on})^3+(U_{off})^3\ ]\ \int c\ dz \quad (11)$$

with: $U_{on}=U_{\delta,f}$= near-bed peak orbital velocity in onshore direction (in wave direction) and $U_{off}=U_{\delta,b}$= near-bed peak orbital velocity in offshore direction (against wave direction), c= time-averaged concentration and γ= phase lag function.

Equation (11) is based on an instantaneous response of the suspended sand concentrations (C) and transport ($q_{s,w}$) to the near-bed orbital velocity (C proportional to U^3 and q_s to U^4). This may be valid for the near-bed layer (say 1 to 5 times the wave boundary layer thickness), but at higher levels a delayed response of the sand concentrations (phase lag effects) will be more realistic, particularly for fine sediments. For very fine sediment the wave-related suspended transport may even be opposite to the wave propagation direction.

Phase lag effects are supposed to be accounted for by the γ-function. As phase lag effects are related to the wave conditions, sand size and bed geometry, the γ-function is supposed to be a complicated function of the former parameters (yielding negative values for very fine sand). A detailed discussion of phase lag effects and functions is given by Dohmen-Janssen (1999).

Modelling of the wave-related suspended transport according to Eq. (11) requires computation of the time-averaged sand concentration profile and integration of the time-averaged sand concentration profile in vertical direction. Herein, the integration is taken over a near-bed layer with a thickness equal to about 0.5 m, assuming that the suspended sand above this layer is not much effected by the high-frequency wave motion with periods in the range of T= 5 to 10 s. This assumption is satisfied if the fall time of a suspended sand particle over a distance of 0.5 m is much larger than the wave period (T_{fall}= 0.5/w_s yielding about 25 s for d= 0.2 mm with w_s= 0.02 m/s). Furthermore, the data of the Deltaflume (Chung and Grasmeijer, 1999) show that most of the wave-related suspended transport occurs in the near-bed layer with a thickness of about 0.5 m (10 to 20 times the ripple height). Chung and Grasmeijer (1999) have determined the γ-function by fitting of Eq (11) to the measured wave-related transport rates. The peak onshore and offshore orbital velocities as well as the time-averaged sand concentrations were taken from measured data in the Deltaflume of Delft Hydraulics with sand of 0.16 and 0.3 mm (see Grasmeijer, 2002). Amazingly, the γ-function was found to be a constant value of about 0.2 for all test results (relative standard error of about 30 %). Any influence of the wave conditions and/or the sand size on the γ-function could not be detected, implying relatively small phase lag effects for the five data sets used. It is noted that the γ-value of 0.2 is based on data with rather pronounced ripples observed in a large-scale 2D wave flume (Delta-flume). The γ-value may be considerably smaller (between 0.1 and 0.2) for field conditions with less pronounced 3D-ripples (Grasmeijer et al., 2000), (Grasmeijer, 2002).
Equation (11) implemented in the TRANSPOR2000 model has been used to compute the wave-related suspended transport for the five Delta flume cases 1A to 1E. The near-bed orbital velocities during the onshore and offshore phase of the wave cycle have been represented by sine-functions based on the measured near-bed peak orbital velocities. The computed values are roughly 1.5 to 2 times the measured values for the 0.16 mm sand. The computed values for the 0.33 mm sand are somewhat too small (about 25%). It should be realized that the measured transport rates have a relatively large inaccuracy range (about factor 2), because the transport rates in the unmeasured zone between z= 0.01 and 0.075m are based on extrapolation.
Equation (11) implemented in the TRANSPOR2000 model has similarly been used to compute the wave-related suspended transport for the field data of the Egmond site 1998 (6 cases). The best results are obtained for γ of about 0.1. Thus, the γ-value representing the Egmond data is considerably smaller (factor 2) than that derived from the data of the Deltaflume, which may be related to the type of bed forms generated in nature and in the 2D wave flume. The bed forms at the Egmond site generally were somewhat longer and flatter than those in the Deltaflume resulting in less pronounced vortex motions and hence smaller wave-related suspended transport rates.

Approximation method

Equation (11) requires numerical computation of the sediment concentration profile.
As an approximation method, the following expression (12) is given for the wave-related suspended transport component (oriented in the wave direction):

$$q_{s,w} = \gamma \; U_A \, L_T \qquad (12)$$

with:

$q_{s,w}$ = wave-related suspended transport (in kg/s/m)
$U_A = [(U_{on})^4-(U_{off})^4]/[(U_{on})^3+(U_{off})^3]$=velocity asymmetry value (m/s),
$L_T = 0.007\ \rho_s\ d_{50}\ M_e$ = suspended sediment load (kg/m^2),
$M_e=(v_{eff}-v_{cr})^2/((s-1)g\ d_{50})$= sediment mobility number due to waves and current (-),
$U_{on}=U_{\delta,f}$= near-bed peak orbital velocity (m/s) in onshore direction (in wave direction) based on significant wave height,
$U_{off}=U_{\delta,b}$= near-bed peak orbital velocity in offshore direction (against wave direction) based on significant wave height,
$v_{eff}=(v_R^2+U_{on}^2)^{0.5}$=effective velocity due to currents and waves
v_R= magnitude of depth-averaged current velocity vector (m/s),
$v_{cr}= 0.19(d_{50})^{0.1}\log(4h/d_{90})$= critical velocity (m/s) for $0.0001<d_{50}<0.0005$ m,
$v_{cr}= 8.5\ (d_{50})^{0.6}\log(4h/d_{90})$= critical velocity (m/s) for $d_{50}\geq 0.0005$ m,
h= water depth (m),
d_{50}, d_{90}= particle size characteristics (in meters),
$s=\rho_s/\rho_w$= relative density (-),
γ= 0.2=phase lag coefficient.
Equation (12) has been fitted to Equation (11). The modified method of Isobe and Horikawa (1982) has been used to compute the peak orbital velocities at the edge of the wave boundary layer (see Grasmeijer and Van Rijn, 1998). Assumptions are: $d_{90}=2d_{50}$, $d_s=d_{50}$=suspended size, $k_{s,w}=k_{s,c}$=bed roughness= 0.03 m, wave-current angle= 90^o, temperature=15 ^{o}C, salinity=30 promille. Three particle sizes have been considered: d_{50}= 0.15, 0.25 and 0.4 mm. The approximation method yields values, which are generally within a factor 2 of those based on the numerical method (Eq. 11).

Current-related suspended transport

Basic formulations

Modelling of the current-related suspended transport requires modelling of both the time-averaged velocity profile and the time-averaged sand concentration profile (Van Rijn, 1987, 1993). The suspended transport in the main current direction is computed as: $q_{s,c,1}= \int v_{R,z}\ c_z\ dz$ with c_z= time-averaged sand concentration at height z and $v_{R,z}$= current velocity at height z in main current direction.
The current velocity profile in the main current direction is represented as a two-layer system to account for the wave effects in the near-bed layer (Van Rijn and Kroon, 1992; Van Rijn, 1993). In both layers the velocity profile is assumed to be logarithmic.
The suspended transport due to the undertow current is computed as: $q_{s,c,2}= \int u_{r,z}\ c_z\ dz$
with c_z= time-averaged sand concentration at height z and $u_{r,z}$= undertow velocity at height z above the bed. The current velocity profile of the undertow current (due to breaking waves), which is opposite to the wave direction, is represented as a three-layer system. In the two layers near the bed the velocity profile is assumed to be logarithmic. In the upper layer (z/h>0.5) the velocity profile is assumed to decrease to zero at the water surface according to a third power distribution.

Both current-related suspended transport components ($q_{s,c,1}$ and $q_{s,c,2}$) are summed by vectorial addition.
The total suspended transport is obtained by vectorial summation of $q_{s,c}$ and $q_{s,w}$.

The time-averaged (over the wave period) advection-diffusion equation is applied to compute the equilibrium time-averaged sand concentration profile due to the combined effect of steady and oscillatory flow. This equation reads as:

$$w_{s,m} + \varepsilon_{s,cw} \frac{dc}{dz} = 0 \qquad (13)$$

in which: $w_{s,m}$ = fall velocity of suspended sediment in a fluid-sediment mixture (m/s), $\varepsilon_{s,cw}$ = sediment mixing coefficient for combined steady and oscillatory flow (m^2/s), c = time-averaged concentration at height z above the bed (kg/m^3); hindered settling and turbulence damping effects are taken into account.

For combined steady and oscillatory flow the sediment mixing coefficient (Fig. 1) is modelled as:

$$\varepsilon_{s,cw} = ((\varepsilon_{s,c})^2 + (\varepsilon_{s,w})^2)^{0.5} \qquad (14)$$

in which: $\varepsilon_{s,w}$= wave-related mixing coefficient (m^2/s), $\varepsilon_{s,c}$= current-related mixing coefficient due to main current and undertow current (m^2/s); the effect of the sediment particles on the mixing of fluid momentum is taken into account by the β_c-factor, which depends on the particle fall velocity of the suspended sand particles and the bed-shear velocity ($\beta_c=1+2(w_s/u_{*,c})^2$ with $\beta_c \leq 1.5$). Thus: $\varepsilon_{s,c}= \beta_c \varepsilon_{f,c}$. Similarly, $\varepsilon_{s,w}= \beta_w \varepsilon_{f,w}$.
Some parameters of the wave-related sediment coefficient have been modified (with respect to the TR1993 model, Van Rijn, 1993) based on analysis of the experimental data. The vertical distribution has not been changed.

$$\delta_s = \max(5\gamma_{br}\, \delta_w,\ 10\gamma_{br}\, k_{s,w}) \text{ with limits } 0.1 \leq \delta_s \leq 0.5 \text{ m} \qquad (15)$$

with: δ_s = thickness of effective near-bed sediment mixing layer, δ_w= thickness of wave boundary layer, $k_{s,w}$= wave-related bed roughness and $\gamma_{br} = 1+ (H_s/h-0.4)^{0.5}$= empirical coefficient related to wave breaking ($\gamma_{br} = 1$ for $H_s/h \leq 0.4$).

The wave-related sediment mixing coefficient in the upper half of the water column has been modified into (multiplied by the breaking coefficient γ_{br}):

$$\varepsilon_{s,w,max}= 0.035\ \gamma_{br}\ h\ H_s/T_p \text{ with } \varepsilon_{s,w,max} \leq 0.05 \text{ m}^2/\text{s}. \qquad (16)$$

The wave-related sediment mixing coefficient near the bed is described by:

$$\varepsilon_{s,w,bed}= 0.018\ \beta_w\ \delta_s\ U_\delta \qquad (17)$$

in which: U_δ = near-bed peak orbital velocity, δ_s= thickness of mixing layer, β_w = coefficient=$1+2(w_s/u_{*,w})^2$ with $\beta_w \leq 1.5$, w_s= fall velocity of suspended sand, $u_{*,w}$= wave-related bed-shear velocity.

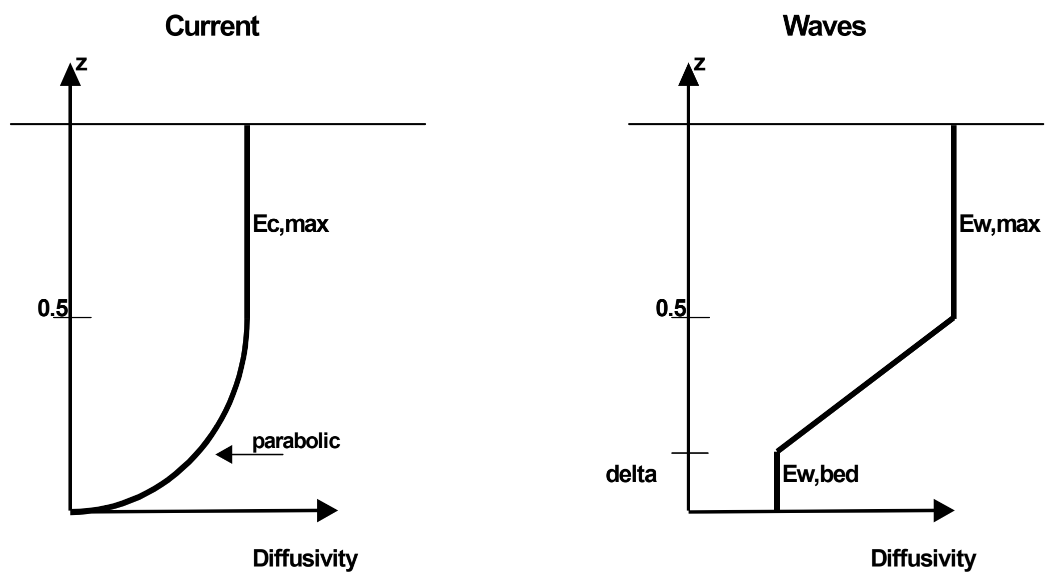

Figure 1 Vertical distribution of mixing coefficients

The near-bed mixing parameter $\varepsilon_{s,w,bed}$ was found to be dependent on the particle velocity (size), based on analysis of sand concentration profiles of experiments with bed material in the range of 0.1 to 0.3 mm (Van Rijn, 1993). The near-bed mixing appears to increase with increasing particle size, which may be an indication of the dominant influence of centrifugal forces acting on the particles due to strong turbulence-induced vortex motions close to the bed resulting in an increase of the effective mixing of sediment particles. This effect is modelled by the β_w-factor. As no information is available for bed materials larger than about 0.3 mm, the application of Eq. (17) for these conditions is highly uncertain. More research is necessary for accurate prediction of the wave-induced suspended transport for relatively coarse materials (>0.3 mm; coarse sand and gravel beds).
Numerical solution of the advection-diffusion equation requires the specification of the concentration at a certain elevation above the bed (reference concentration, see Fig. 2).
The reference concentration (volume) is given by:

$$c_a = 0.015 \ \frac{d_{50}}{a} \ \frac{T^{1.5}}{D_*^{0.3}} \quad \text{with } c_a \leq 0.05 \text{ (approx. } 130 \text{ kg/m}^3\text{)} \tag{18}$$

in which: D_*= dimensionless particle parameter (-), T= dimensionless bed-shear stress parameter (-), a = reference level (m), a is taken equal to the bed roughness k_s with a minimum value of 0.02 m.

The T-parameter is:

$$T = (\tau'_{b,cw} - \tau_{b,cr}) / \tau_{b,cr} \tag{19}$$

in which: $\tau'_{b,cw}$= time-averaged effective bed-shear stress (N/m²), $\tau_{b,cr}$= time-averaged critical bed-shear stress according to Shields (N/m²).

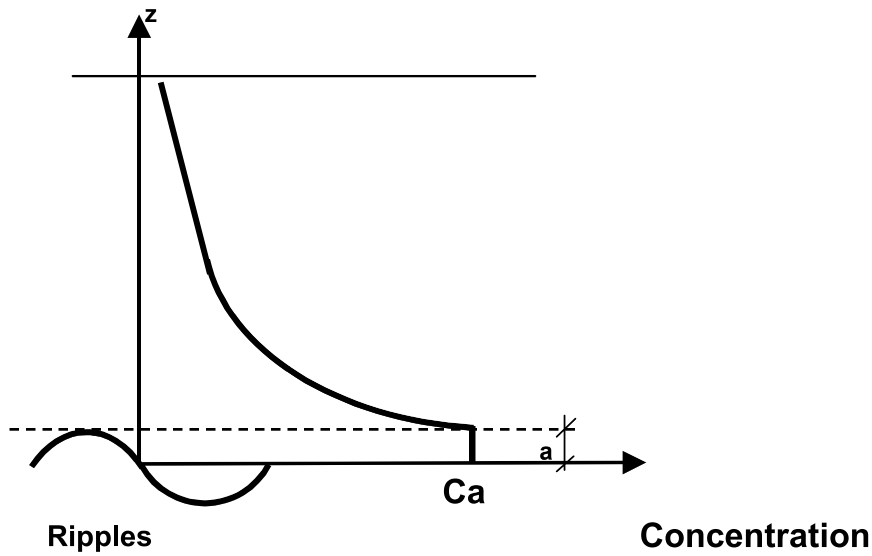

Figure 2 Reference concentration

The magnitude of the time-averaged bed-shear stress, which is independent of the angle between the wave- and current direction, is given by:

$$\tau'_{b,cw} = \tau'_{b,c} + \tau'_{b,w} \tag{20}$$

in which: $\tau'_{b,c} = \mu_c\, \alpha_{cw}\, \tau_{b,c}$= effective current-related bed-shear stress (N/m²), and $\tau'_{b,w} = \mu_{w,a}\, \tau_{b,w}$ = effective wave-related bed-shear stress (N/m²), μ_c= current-related efficiency factor, $\mu_{w,a}$= wave-related efficiency factor and α_{cw}= wave-current interaction factor; grain-related friction factor depends on d_{90}.

The wave-related efficiency factor $\mu_{w,a}$ is an important parameter, because it strongly affects the reference concentration near the bed. This parameter will probably depend on the bed form and bed roughness characteristics, but the functional relationship involved is not yet known. Therefore, the $\mu_{w,a}$ factor has been used as a calibration parameter to get a better estimate of the near-bed concentration. As the bed forms are related to the relative wave height (ripples for small values of H_s/h and plane bed for large values of H_s/h), the $\mu_{w,a}$ factor is supposed to be related to the relative wave height. Based on analysis of experimental data, the $\mu_{w,a}$ factor has been modified into $\mu_{w,a} = 0.125(1.5\text{-}\ H_s/h)^2$ with minimum value of 0.063. This yields a better description of the reference concentration for relatively small wave heights in the ripple regime compared with the earlier method (TR1993, Van Rijn, 1993).
Important parameters for the suspended load transport are the current-related and the wave-related bed-form roughness ($k_{s,c}$ and $k_{s,w}$). These parameters are directly related to the size and geometry of the bed forms (ripples). The wave-related bed-form roughness is also related to the near-bed orbital excursion. If the ripple length is much larger than the orbital excursion, the wave-related bed-form roughness is relatively small because the ripple-related vortices will be relatively weak. At present stage of research both parameters ($k_{s,c}$ and $k_{s,w}$) are used as input parameters.

Approximation method

The computation of suspended sand transport for combined current and wave conditions is based on numerical integration of diffusion equation for the sand particles, yielding the sand concentration profile. The current-related suspended sand transport (oriented in the direction of the depth-averaged velocity vector) can be determined from:

$$q_{s,c} = \int v_R c \, dz \tag{21}$$

Van Rijn (1993) has introduced an approximation method for the current-related suspended sand transport in steady flow conditions (in direction of velocity vector), as follows:

$$q_{s,c} = F_c \, v_R \, h \, c_a \tag{22}$$

where: $q_{s,c}$= current-related suspended transport (in kg/s/m, if concentration in kg/m^3), v_R=depth-averaged current velocity (magnitude of vector), h=water depth, c_a= reference concentration (kg/m^3), F_c=correction factor (Van Rijn, 1984b).

The correction factor F_c is described by:

$$F_c=[(a/h)^{ZC}-(a/h)^{1.2}]/[(1.2-ZC)\,(1-a/h)^{ZC})] \tag{23}$$

$$ZC=w_s/(\beta_c\,\kappa\,u_{*,c})+2.5(w_s/u_{*,c})^{0.8}(c_a/c_o)^{0.4} \tag{24}$$

where: $u_{*,c}$= current-related bed-shear velocity, c_a= reference concentration, c_o= maximum bed concentration=0.65 (volume), w_s= fall velocity (based on d_{50} of bed material), β_c=1+2$(w_s/u_{*,c})^2$ with $\beta_{c,max}$=1.5.

For conditions with currents and waves this method can be generalised, as follows:

$$q_{s,c} = (F_c + F_w) \, v_R \, h \, c_a \tag{25}$$

The correction factor F_w is described by:

$$F_w=[(a/h)^{ZW}-(a/h)^{1.2}]/[(1.2-ZW)\,(1-a/h)^{ZW})] \tag{26}$$

The ZW parameter of the approximation method has been determined by computer fitting based on a set of numerically computed suspended transport rates, yielding:

$$ZW=4\,(h/h_{ref})^{0.6}\,(w_s\,T_p/H_s)^{0.8} \tag{27}$$

where: h=water depth, h_{ref}= reference water depth (= 5 m), T_p= peak period of waves, H_s=significant wave height, w_s= fall velocity ($w_{s,minimum}$= 0.03 m/s).

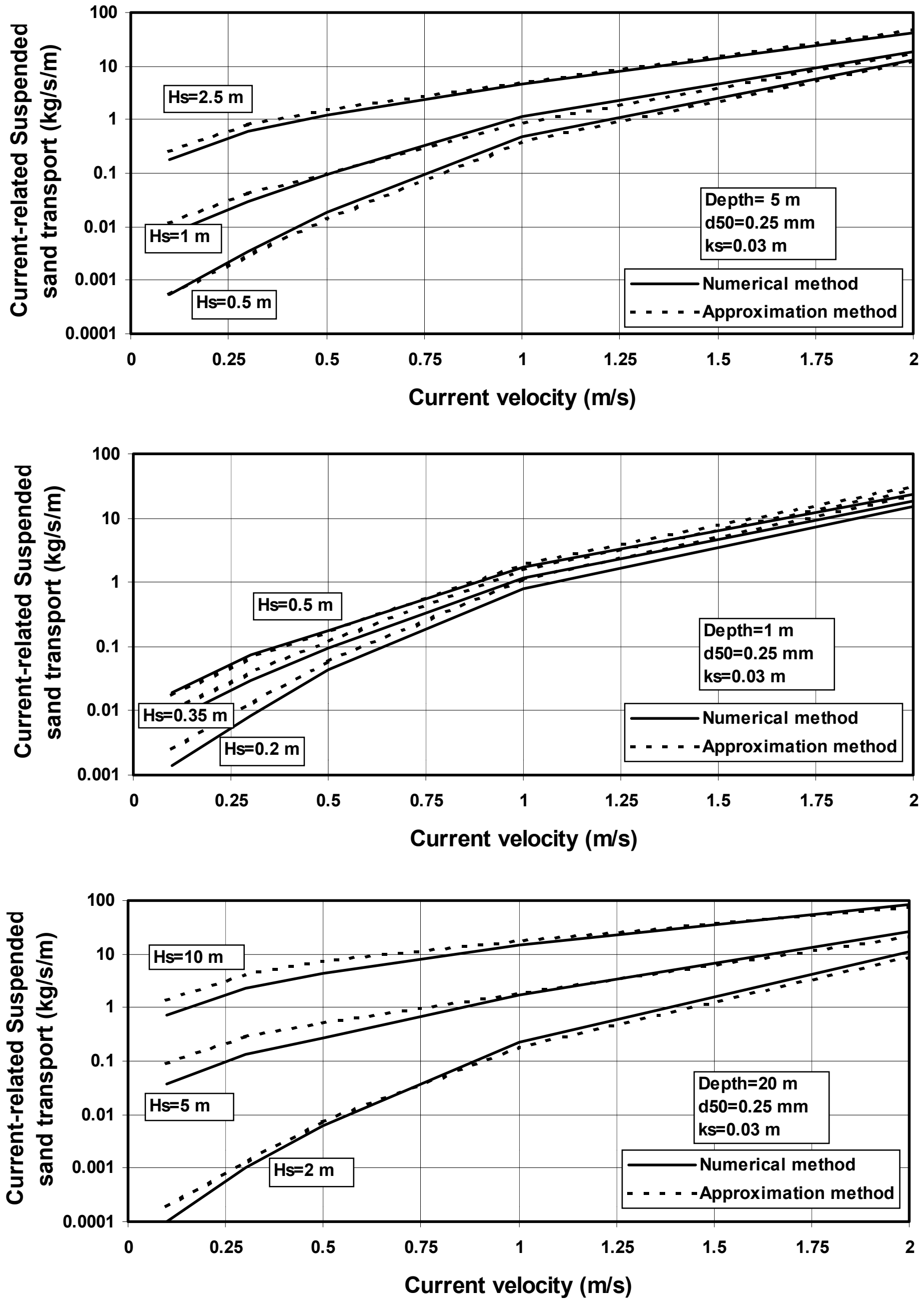

Figure 3 Current-related suspended transport based on numerical method (TR2000 model) and approximation method for d_{50}=0.25 mm; h=1, 5 and 20 m

The numerical method (TR2000) has been used to determine the suspended transport rates for the following conditions (water temperature= 15 degrees C, Salinity= 30 promille): d_{50}=0.15, 0.25 and 0.4 mm, $d_{90}=2d_{50}$, water depths=0.25, 1, 5, 20 and 50 m, $k_{s,c}= k_{s,w}$= 0.03 m, $d_s=d_{50}$ of bed material and wave-current angle= 90 ^{o} (see Van Rijn, 2001).
The approximation results for d_{50}= 0.25 mm are shown in Figure 3. Generally, the results of both methods are within a factor of 2 of each other. More results are given by Van Rijn (2001).

CONCLUSIONS

Simple approximation formulae have been developed to represent (with an accuracy of about a factor of 2) the bed load and suspended load transport under combined current and wave conditions, as described by the detailed TRANSPOR2000 model.

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