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EFFECT OF BASIC MUD PARAMETERS ON COMPUTED CONCENTRATION AND TRANSPORT RATES

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ABSTRACT

The parameters which quantify the settling rates and sediment bed concentrations are tested in a time dependent mud transport numerical model. The effect of flocculation on the concentration distributions and transport are examined. The inclusion of flocculation is found to have a significant effect on concentration distributions and transport rates. A specified sediment bed concentration boundary condition is compared to a flux boundary condition.

INTRODUCTION

A three-dimensional (3-D) transport model developed by Van Rijn and Meijer (1986) called SUSTRA has been applied to mud transport across a navigation channel by Toro et al (1989). Transport over the navigation trench was based upon assumed typical values for mud settling and suspension. The same basic approach will be applied herein. The emphasis will be upon the effect on predictions of variations in the assumed values. The model has been described in detail in the previous references, so the model will be described only briefly. The following results were part of a larger effort to examine the sensitivity and reliability of the SUSTRA model.

The 3-D, transient (in time t) transport equation solved by the model is

$$\frac{\partial c}{\partial t} + \frac{\partial}{\partial x}(uc) + \frac{\partial}{\partial y}(vc) + \frac{\partial}{\partial z}[(w-w_s)c] = \frac{\partial}{\partial x}\left(\epsilon_x \frac{\partial c}{\partial x}\right) + \frac{\partial}{\partial y}\left(\epsilon_y \frac{\partial c}{\partial y}\right) + \frac{\partial}{\partial z}\left(\epsilon_z \frac{\partial c}{\partial z}\right) \qquad (1)$$

where c is the local mud concentration, u, v and w are the velocities in the x, y and z coordinate directions respectively, and w_s is the local settling rate of the sediment. The turbulent redistribution is modeled with a standard flux-gradient expression in which ϵ_z is the vertical diffusivity and ϵ_x and ϵ_y are the horizontal diffusivities. The boundary conditions for the equation are specified according to known inflow and outflow conditions, no flux at any water free surface points, and bed boundary conditions determined from an empirical transport stage parameter T. The stage parameter is defined in terms of the critical bed erosion stress $\tau_{b,cr}$ and the actual bed stress in the form

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$$T = \frac{\tau_b - \tau_{b,cr}}{\tau_{b,cr}} \tag{2}$$

The actual bed stress τ_b is determined by a roughness expression of the form

$$\tau_b = \rho g \left[\frac{\bar{u}}{C_k} \right]^2 \tag{3}$$

C_k is a Chezy roughness coefficient and is determined in terms of the equivalent sand grain roughness for currents k_{sc} as

$$C_k = 18 \log \left[\frac{12\,H}{k_{sc}} \right] \tag{4}$$

The bed boundary condition is then described by either of two relations:

$$c_{bed} = \frac{MT}{w_{s\,0}} \qquad \textit{Specified concentration} \tag{5}$$

$$\left[\frac{\partial c}{\partial z} \right]_{bed} = M'\,T \qquad \textit{Specified flux} \tag{6}$$

in which M and M' are the respective specified flux or concentration fitting constants for mud, and w_{s0} is the clear water (or non-flocculent) settling velocity of mud particles).

The fall velocity of the sediment is described by a flocculation expression of the form

$$w_s = \alpha\ w_{s\,0}\ c^{\beta} \tag{7}$$

where α and ß are empirical constants. As noted by van Rijn (1990), this expression should be valid for mud concentrations below 10,000 mg./L. The power coefficient ß will be positive and greater than 1 in this range, such that the net settling rate increases with increasing concentration.

All computations will utilize the flow geometry and temporal flow description set out in the tidally induced time varying non-uniform flow over a trench reported by Toro et al. (1989). The geometry of the system is shown in Figure 1. The flow, driven by tidal variation, traverses the deepened section (referred to as the trench) in either the positive or negative x direction only. Even though the flow direction changes during tidal variation, the left-hand side of the trench will be referred to as the upstream end, because flow initially moves from that end toward the trench. The extent of the computation region is prescribed as 6500 m. to approximate the distance required to fully develop the concentration distribution. The time mean depth of the water flow is set at 10 m., although the depth varies due to tidal variations. The trench provides an additional 10 m. of depth over a width of 500 m., and has sloping side walls on either side rising at a mild slope of 1 to 50 back to the level of the non-trench flow regions.

The flow variation in time is set according to a simple sinusoidal variation

$$Q = Q_0 \sin \left(\frac{2\pi t}{T} \right) \tag{8}$$

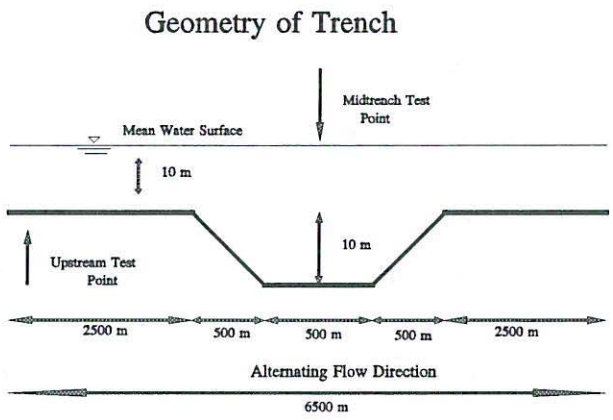

Figure 1. Geometry of the Test System

where Q_0 is the maximum flow and T is the tidal period. The tidal angle ($\theta = 2\pi t/T$) varies from 0 to 2π over the tidal period. In terms of degree angles, the maximum flows occur at $\theta = 90°$ and 270°. Also, the depth averaged velocities are not the same at 90° and 270°, because water depth varies according to a lagged sinusoidal expression of a form similar to the flow variation expression, and is

$$H = H_0 + h_0 \sin\left(\frac{2\pi t}{T} + \psi\right) \tag{9}$$

where H_0 is the time mean water surface depth upstream as shown in Figure 1, h_0 is the amplitude of the depth variation, and ψ is a phase lag between maximum flow depth and maximum flow. The instantaneous depth averaged velocity U is simply Q/H and has no component in the y direction. The variation of U in the vertical is prescribed by a log-law relation as

$$u = \frac{U \ln\left(\frac{z}{z_0}\right)}{\left[\frac{z_0}{H} - 1 + \ln\left(\frac{H}{z_0}\right)\right]} \tag{10}$$

where z_0 is the zero velocity level which should occur at 0.033 k_{sc}. The vertical velocity variation is applied by enforcing continuity based on the prescribed variations of u and v.

There are several settings which are implicitly assumed in the model related to initial and boundary conditions. The model sets all initial values of concentration to the steady flow, instantaneous equilibrium conditions (IE) determined by the initial flow conditions. In a similar fashion, any inflow and outflow boundary conditions (I/O B.C.'s) are set to IE conditions at every time step. It is assumed that the non-trench region is sufficiently large to justify this assumption. Other important model parameters are the time step Δt, and the x and y sediment diffusivities e_x and e_y. The values of various parameters and model assumptions used in for the reference case are reported in Toro et al. (1989) as:

Q_0 = 10 m^3/sec	ψ = 30°	T = 12 hours
H_0 = 10 m	k_{sc} = 0.01 m	Δt = 20 minutes
e_x = 0.5 m^2/sec, e_y = 0	$\tau_{b,c}$ = 0.1 N/m^2	h_0 = 2 m
w_{s0} = 10^{-4} m/sec	α = 2.82x10^5	ß = 1.3
M = 4x10^{-8} m/sec	z_a = 0.01 m	I.C.'s : IE
I/O B.C.'s : IE	Bed B.C.'s: Specified c	

RESULTS

The reference solution is represented by plotting three points in the vertical over 4 tidal cycles. Figure 2 shows the variation at the upstream point, while Figure 3 shows the same variation in the trench. The concentrations appear to drop off much more quickly than might be expected given the small value of w_{s0}, and one might expect more uniform concentrations with depth with such a low settling rate. For the upstream plot, the concentration drops off very rapidly from the bottom concentration. The curve labeled as a middle level concentration is actually somewhat less than one half of the distance from the bed, so by the relatively similar magnitude of the surface concentration, it is apparent that most of the drop in concentration occurs near the bed. The midtrench concentrations exhibit a more uniform distribution of concentration over the depth around the 1500 mg./L. level. The uniformity of concentration in the trench can be explained by two effects. First, convection carries sediment into the trench mainly from the upper layers. Not surprisingly then, the concentrations away from the bed upstream are on the order of 1500 mg./L. The second effect is the lowering of the bed

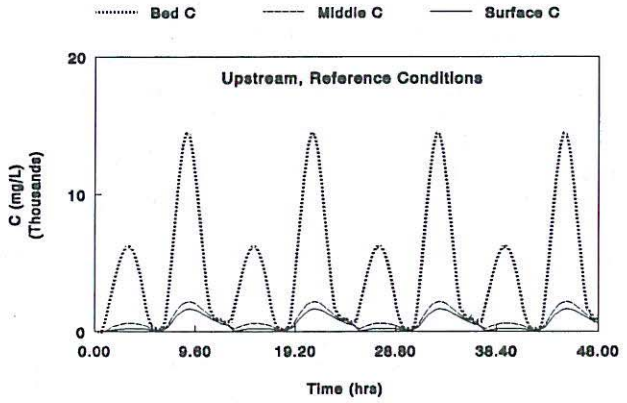

Figure 2. Upstream Concentrations, Reference Conditions

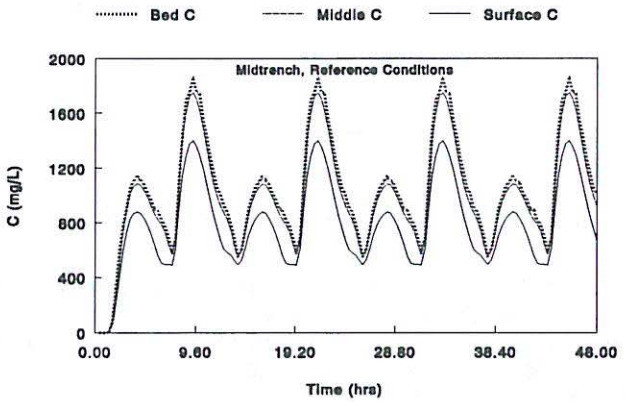

Figure 3. Midtrench Concentrations, Reference Conditions

concentration in the trench. This is due to the fact that the mean velocity has slowed to one half its value upstream. The T-parameter is related to velocity squared, and the relatively rapid response of the bed concentrations are due to the deceleration.

The explanation to the apparently rapid response to changes in the bed concentration and the steep drops in concentration upstream can be explained in terms of the flocculent settling conditions chosen. For the values of α, ß, and w_s in the reference case, the following variation in settling velocity with concentration results.

w_s = 0.034 m/sec for C = 15,000 mg/L
w_s = 0.02 m/sec for C = 10,000 mg/L
w_s = 0.008 m/sec for C = 5,000 mg/L
w_s = 0.001 m/sec for C = 1,000 mg/L

The first concentration is on the order of the maximum bed concentrations, and the last is on the order of the maximum free surface concentrations. This causes the sediment to settle at about the rate of sand over most of the depth, and explains the sharp concentration gradients near the bed.

The sediment transport rate produced by these distributions is now examined. Also, the relative importance of the bed boundary condition chosen is studied. Figures 4 and 5 are plots of the transport variation with time using both the bed concentration boundary condition of Eq. (5) and the flux condition of Eq. (6). As might be expected with the similarity of the two curves, the concentration plots for the flux boundary condition application at the bed were similar to those in Figures 2 and 3. The transport rates were determined by numerically integrating the concentration profiles and velocities over the depth. It is important to note

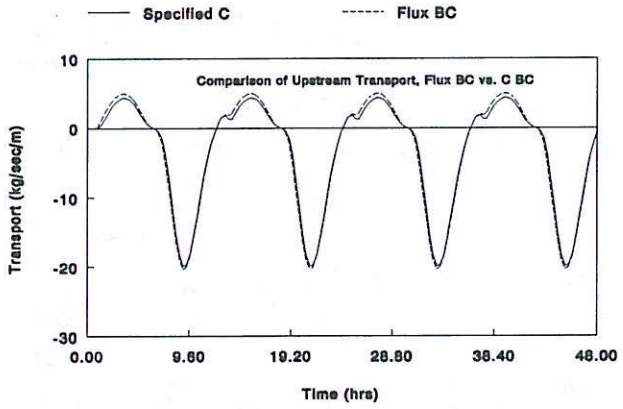

Figure 4. Upstream Transport, Reference Conditions

however that the value of M' was calibrated to produce similar maximum bed concentrations to the bed specified concentration B.C. case. The value of M' which was used was 6.3×10^{-7}. In actual field applications, M or M' would need to be calibrated to known values of concentration, so the calibration is reasonable. The important trend to note in comparing upstream to midtrench transport is that the maximum and minimum values are on the same order of magnitude. This is indicative of the importance of the transport of upper level sediment upstream into the trench and the lesser importance of the bed concentrations.

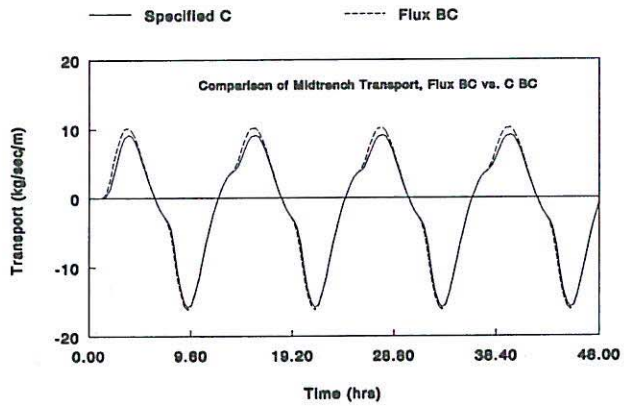

Figure 5. Midtrench Transport, Reference Conditions

With the fall velocity exhibiting such an important effect on concentration distributions, the effect of variation in fall velocity was examined. Figures 6 and 7 are plots of upstream and mid-trench distributions as a function of time without any flocculation effects. The settling rate was taken to be a representative value of discrete particle settling velocity for mud (10^{-4} m./sec.), and all other model parameters were held the same as in the reference case. The concentrations show very little variation over the entire depth, because the lack of any flocculation leads to an absence of the sharp gradients near the bed seen in the reference calculation. Mid-trench concentrations increase dramatically from the reference solution as convection is allowed to sweep the slowly settling sediment into the trench with little settling.

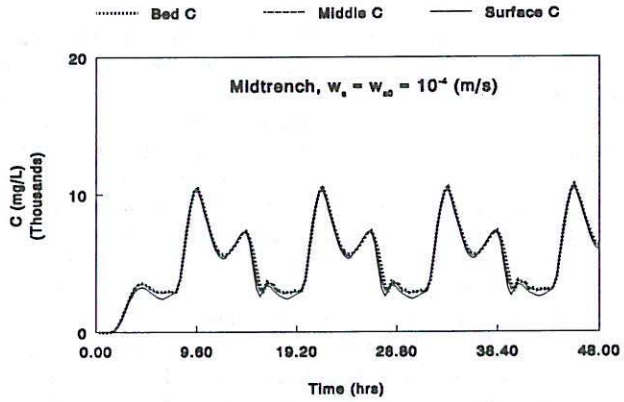

Figure 6. Midtrench Conc., No Flocculation Effect, $w_{s0} = 10^{-4}$ m./sec.

An interesting effect of the model's handling of bed concentrations can be noted clearly in the mid-trench predictions, where the concentrations are nearly all the same with depth at any time. This is the result of the fact that bed concentrations are never allowed to be lower than the nodal concentration above the bed. Concentration increasing with distance away from the bed would be very unrealistic under the deposition conditions experienced in the trench, yet

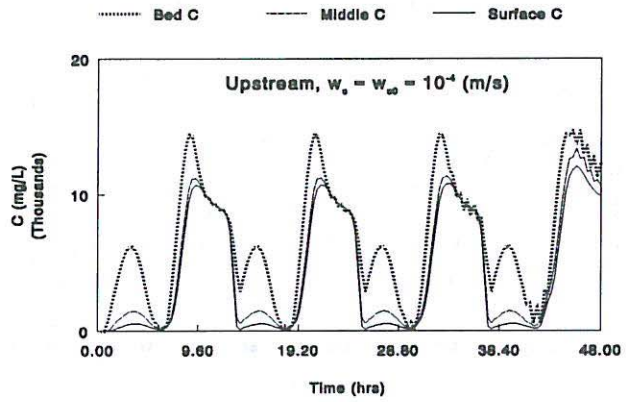

Figure 7. Upstream Conc., No Flocculation Effects, $w_{s0} = 10^{-4}$ m. sec.

the prescribed concentration boundary condition can sometimes attempt to force the concentration at the bed below those above. Application of this limit can be seen in those portions where all concentrations are nearly the same with depth. The transport for the constant fall velocity prediction (not shown) exhibits dramatic increases, especially during the reversed flow periods where transport is over six times as large as in the reference case. The higher transport rates in the reversed flow periods are related to the phase lag between the tidal flow and the depth. This causes higher velocities in the reversed flow periods as depth decreases during increasing flow. The higher velocity is apparent in that this is the period of highest bed concentrations as well.

While it appears that modeling mud transport with a constant discrete particle fall velocity produces unrealistic results, there can be considerable variation in the flocculent fall velocity range. Van Rijn (1990) shows flocculent settling data for different mud samples which vary in settling rates by an order of magnitude over the entire concentration range. Furthermore, flocculation effects can increase the effective settling velocity over most of the depth to an order of magnitude increase over discrete settling values, as demonstrated earlier. Figures 8 and 9 are the result of a model prediction approximating flocculent settling rates increasing w_{s0} by a factor of 10 to 10^{-3} m./sec, but holding settling constant with respect to concentration as in the previous test. In order to keep a reasonable basis of comparison to the reference case, the M value for the specified concentration bed boundary condition was increased by a factor of 10 so that the upstream bed concentrations would follow the reference case. This can be clearly seen through Eq. (5) where bed concentration is a function of M/w_{s0}.

The apparent differences in concentration exhibited in Figures 8 and 9 are in the increased values of the concentrations higher into the flow in comparison to the reference case. The higher concentration profiles in the vertical are due to the lack of the steep gradient away from the bed produced by the higher flocculent settling rates near the bed. The resultant transport rate curves do not differ much from the reference case in shape, but in general are a factor of 3 higher at all times. Thus, the use of a higher discrete settling velocity to approximate flocculation effects leads to significant inaccuracies.

ACKNOWLEDGEMENT

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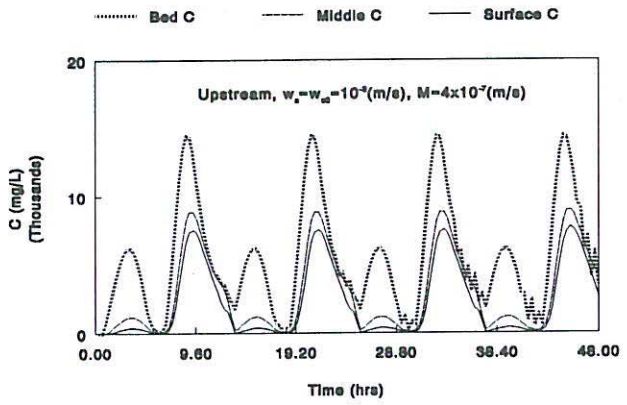

Figure 8. Upstream Concentration, Non-Flocculent, $w_{s0} = 10^{-3}$ m./sec., M = $4x10^{-7}$ m./sec.

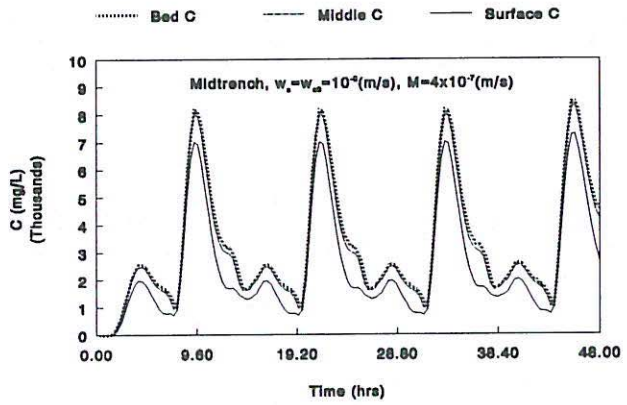

Figure 9. Midtrench Concentration, Non-Flocculent, $w_{s0} = 10^{-3}$ m./sec., M = $4x10^{-7}$ m./sec.

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