2-D Vertical and 3-D Modelling of Mud Transport in Tidal Flows

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Abstract

Model predictions of a fully three-dimensional sediment transport model are presented for two tidal flow variation cases. A simplified two-dimensional form of the model is applied to demonstrate the effects of inflow and outflow boundary conditions. The full model is applied to predict transport of mud for a coastal channel flow.

Introduction

A two- and three-dimensional (2- and 3-D) transport model developed by Van Rijn and Meijer (1986) called SUSTRA has been applied to mud and sand transport. The application of a 3-D flow and sediment transport models and the analysis of the computed results usually involves significant effort. Often, the available budget is not sufficient to make as many runs as necessary to study the influence of the basic input parameters and boundary conditions. Such a sensitivity study can be carried out relatively easily by using a two-dimensional vertical model, as presented by DeVantier and Van Rijn (1993). Herein, the attention is initially focussed on the influence of the boundary condition at the inlet on the computed results. Finally, an example of the full 3-D model is shown.

The three-dimensional model has been described in detail in the previous references, so the model will be described only briefly. The following results were part of a larger effort to examine the sensitivity and reliability of the SUSTRA model. The 3-D, transient (in time t) transport equation solved by the model is

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$$\frac{\partial c}{\partial t}+\frac{\partial}{\partial x}(uc)+\frac{\partial}{\partial y}(vc)+\frac{\partial}{\partial z}[(w-w_s)c]=\frac{\partial}{\partial x}\left(\epsilon_x\frac{\partial c}{\partial x}\right)+\frac{\partial}{\partial y}\left(\epsilon_y\frac{\partial c}{\partial y}\right)+\frac{\partial}{\partial z}\left(\epsilon_z\frac{\partial c}{\partial z}\right) \quad (1)$$

where c is the total mud concentration, u, v and w are the velocities in the x, y and z coordinate directions respectively, and w_s is the local settling rate of the sediment. The turbulent redistribution is modelled with a standard flux-gradient expression in which ϵ_z is the vertical diffusivity and ϵ_x and ϵ_y are the horizontal diffusivities. The boundary conditions for the equation are specified according to known inflow and outflow conditions, no flux at any water free surface points, and bed boundary conditions determined from an empirical transport stage parameter T. The stage parameter is defined in terms of the critical bed erosion stress $\tau_{b,cr}$ and the actual bed stress τ_b in the form $T = (\tau_b-\tau_{b,cr})/\tau_{b,cr}$. The bed boundary condition is then described by either of two relations:

$$c_{bed} = \frac{MT}{w_{s\,0}} \qquad \textit{Specified concentration} \quad (2)$$

$$\left[\frac{\partial c}{\partial z}\right]_{bed} = M'\,T \qquad \textit{Specified flux} \quad (3)$$

in which M and M' are the respective specified flux or concentration fitting constants for mud, and w_{s0} is the clear water (or non-flocculent settling velocity of mud particles). In these calculations, the fall velocity of the sediment is described by a flocculation expression of the form

$$w_s = \alpha\ w_{s\,0}\ c^{\beta} \quad (4)$$

where α and ß are empirical constants. As noted by Van Rijn (1993), this expression should be valid for mud concentrations below 10,000 mg/l. The power coefficient ß will be positive and greater than 1 in this range, such that the net settling rate increases with increasing concentration. A constant fall velocity is obtained when ß = 0.

The two-dimensional vertical model neglects the $\partial/\partial y$-terms of Eq. (1). All 2DV-computations will utilize the flow geometry and temporal flow descriptions set out in the tidally induced time varying non-uniform flow over a trench. The geometry of the system is shown in Fig. 1. The flow, driven by tidal variation, traverses the deepened section (referred to as the trench) in either the positive or negative x direction only. Even though the flow direction changes during tidal variation, the left-hand side of the trench will be referred to as the upstream end, because flow initially moves from that end toward the trench. The flow variation in time is set according to a simple sinusoidal variation. The water depth varies according to a lagged sinusoidal expression (phase shift = 30°). The vertical velocity distribution is prescribed by a log-law expression using a zero-velocity level of 0.01 m, see DeVantier and Van Rijn (1993).

There are several settings which are implicitly assumed in the model related to initial and boundary conditions. The model sets all initial values of

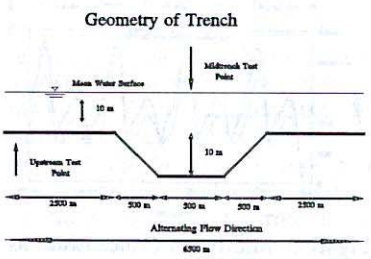

Figure 1. Geometry of the Test System

concentration to the steady flow, instantaneous equilibrium conditions (IE) determined by the initial flow conditions. In a similar fashion, any inflow and outflow boundary conditions (I/O B.C.'s) are set to IE conditions at every time step. It is assumed that the non-trench region is sufficiently large to justify this assumption. Other important model parameters are the time step Δt, and the x and y sediment diffusivities e_x and e_y. The values of various parameters and model assumptions used in the reference case are reported by DeVantier and Van Rijn (1993).

2DV Computations

The reference solution is represented by plotting three points in the vertical over 4 tidal cycles. Figs. 2 and 3 show the variation at the upstream point and in the trench. The concentrations appear to drop off much more quickly than might be expected given the small value of w_{s0}, and one might expect more uniform concentrations with depth with such a low settling rate. For the upstream plot, the concentration drops off very rapidly from the bottom concentration. The curve labeled as a middle level concentration is actually somewhat less than one half of the distance from the bed, so by the relatively similar magnitude of the surface concentration, it is apparent that most of the drop in concentration occurs near the bed. The midtrench concentrations exhibit a more uniform distribution of concentration over the depth around the 1500 mg/l level. The uniformity of concentration in the trench can be explained by two effects. First, convection carries sediment into the trench mainly from the upper layers. Not surprisingly then, the concentrations away from the bed upstream are on the order of 1500 mg/l. The second effect is the lowering of the bed concentration in the trench. This is due to the fact that the mean velocity has slowed to one half its value upstream. The T-parameter is related to velocity squared, and the relatively rapid response of the bed concentrations are due to the deceleration. The explanation of the apparently rapid response to changes in the bed concentration and the steep drops in concentration upstream can be expressed in terms of the flocculent settling conditions chosen. For the values of α, ß, and w_{s0} in the reference case, the settling velocity can vary from ws = 0.034 m/s for c = 15,000 mg/l to ws = 0.001 m/s for c = 1,000 mg.l.

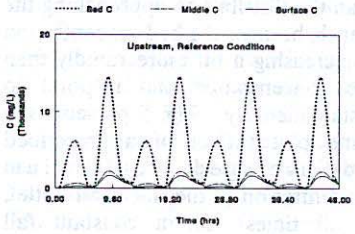

Figure 2. Upstream Concentrations, Reference Conditions

Now the influence of the initial conditions and the boundary conditions at

the inlet and outlet will be examined. Runs were made in which the SUSTRA-model was modified so that an initial condition was applied with an equilibrium profile using the tide-averaged bed concentration of 4800 mg/l upstream in the reference case. Nothing else was different from the reference case. The predictions only differ from the reference case in the first cycle, indicating that the initial condition had no significant effect on the cyclic solution.

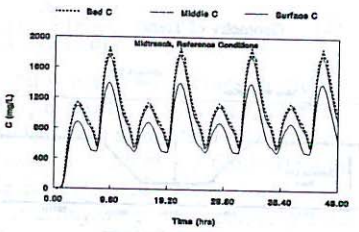

Figure 3. Midtrench Concentrations, Reference Conditions

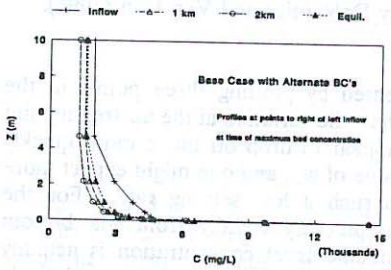

Figure 4. Conc. Profiles, Upstream, Ref. Case, Alternate I.O. B.C.'s.

The possibility that inflow and outflow boundary conditions were leading too quick a response because of convection, was also examined. The same equilibrium conditions as prescribed initially for the whole domain were now used at the inlet and outlet of the computational domain for all times. Surprisingly, the results for upstream and mid-trench concentrations were nearly indistinguishable from the reference case results. The reason can be seen by examining the vertical concentration profile development with distance downstream of the channel at the time of maximum bed concentration shown in Fig. 4.

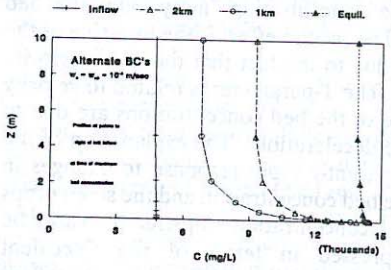

Figure 5. Conc. Profiles, Upstream, Const. Fall Vel., Alternate I.O. B.C.'s.

The boundary condition changes very rapidly to the 2 km distribution from the prescribed value at the inlet. The IE profile is also plotted, and it is somewhat higher than the distribution approaching the trench, because the bed concentration is increasing a bit more rapidly than the concentration can respond to instantaneously. Fig. 5 presents the same plot (effect of a prescribed tide-averaged equilibrium concentration at the inlet and outlet, at all times), for a constant fall velocity, w_s = 0.0001 m/s (no flocculation effects). The results are significantly different.

The I/O B.C.'s have a significant effect, and the low fall velocity means

that the concentration changes so slowly that instantaneous equilibrium is also a poor estimate of conditions. Figs. 6 and 7 show a significant difference in time response compared to the same case without the alternate I/O B.C.'s (Figs 2 and 3). The lowest concentrations are bounded upward by the bed concentration of the inflow of approximately 5000 mg/L (this was 3000 mg/l in the reference case with the same constant fall velocity). The low settling rate and high mixing simply keep the concentration from ever going much below the inflow. To improve on this, the location of the inlet and outlet boundaries in tidal flow must be chosen very far away from the area of interest in case of a small fall velocity (≈ 0.0001 m/s).

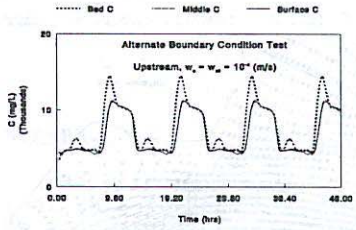

Figure 6. Conc. in Time at Upstream pt., Const. Fall Vel., Alt. I.O. B.C.'s.

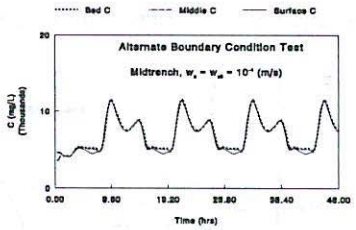

Figure 7. Conc. in Time in Trench, Const. Fall Vel., Alt. I.O. B.C.'s.

3-D Computation

As a part of the proposed new port development at Belview, Waterford, Ireland, it was necessary to increase the depth of the navigation channel to -6.0 m C.D. This requires cutting of channels through the Cheekpoint Upper and Lower Bars (present levels are -3.5 m C.D.). The area of interest is the shipping channel at Upper and Lower Bar near Cheek-point in the river Suir at the confluence with the river Barrow (see Fig. 7). One of the alternative layouts studied was the construction of short groynes (surface level at 2.5 m C.D.) at the south bank in combination with a training wall at the north bank to increase the flow velocities in the Upper and Lower Bar channels (dredged to -6.0 m C.D.) in order to reduce the deposition rates. The channel width is 100 m. The tidal currents were computed by numerical solution of the depth-averaged flow equations. The model covered an area of approximately 7 km^2 of the rivers Suir and Barrow. The presence of the groynes in combination with the training wall yields an increase of the velocities of about 15% at Lower Bar Channel.

The SUSTRA-model was used to compute the sediment transport rates of silt material (30 μm). The vertical distribution of the flow velocity was represented by a log-law expression. The input values were: fluid density = 1023 kg/m^3, sediment density = 2650 kg/m^3, critical bed-shear stress = 0.1 N/m^2, bed roughness = 0.03 m, reference level at bed = 0.03 m, and time step = 30 min. The fall velocity formula used was: $w_s = 1.2x10^5\ w_{s,0}\ c_v$, in which $w_{s,0}$ = 0.03 mm/s and c_v = volume concentration giving values of w_s = 0.068 mm/s for c = 50 mg/l and w_s = 0.68 mm/s for c = 500 mg/l. The mud constant M was calibrated using measured deposition volumes of Upper and Lower Bar channels

during the period November 1989 to May 1990 (with resultant M = 4.10-10). Fig. 7 shows the computed sediment transport rates at maximum ebb flow during springtide conditions. The largest values occur at the location of the channels near the groynes. Circulation patterns can be observed between the groynes and downstream of the groynes (typical deposition areas). Strong erosion does occur near the heads of the groynes where the flow velocities are relatively high. The silt concentrations varied from 500 mg/l just after maximum ebb flow to about 5 mg/l after turning of the horizontal tide. The concentration profiles are almost uniform in the vertical direction.

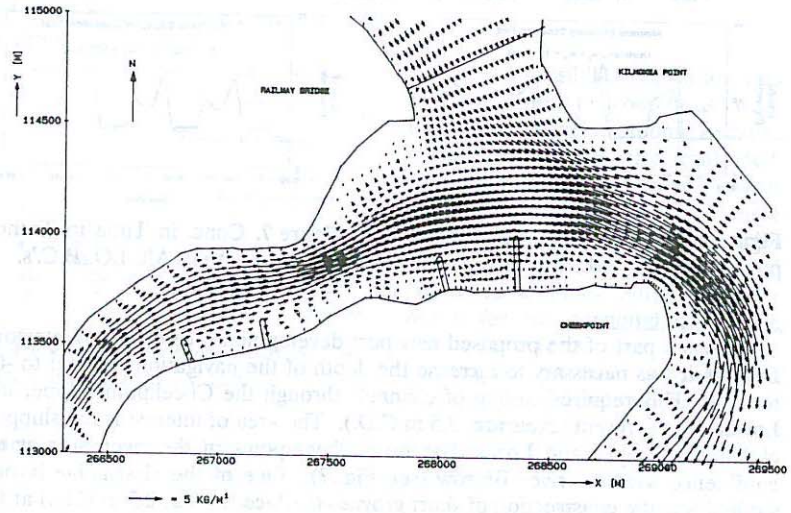

Figure 7 Mud transport rates at maximum ebb flow, spring tide

Acknowledgement

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Appendix. References

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