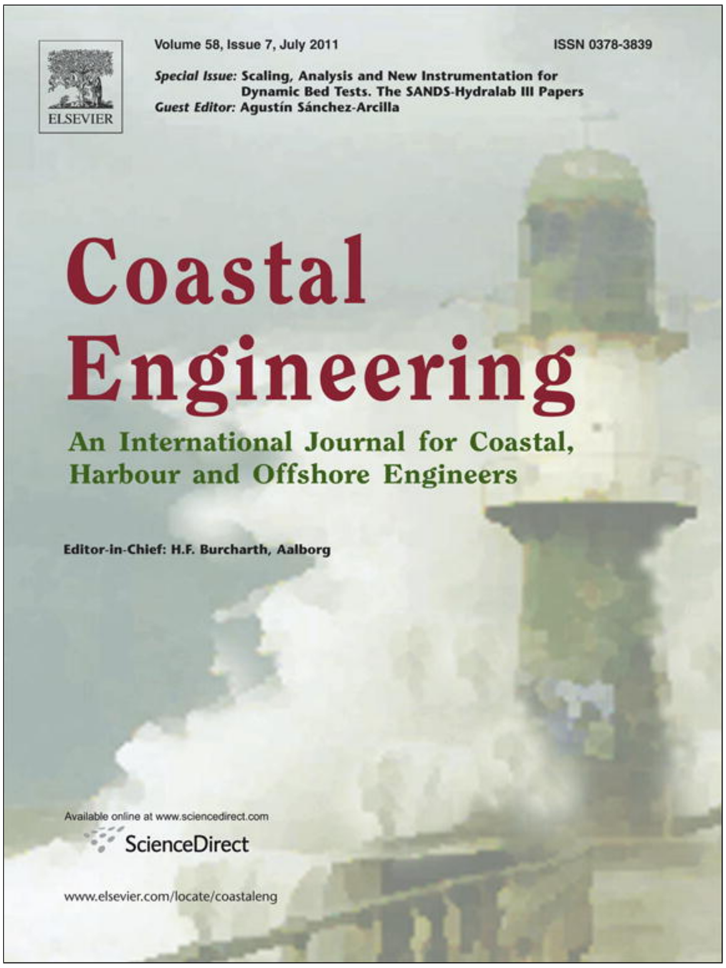
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Scaling laws for beach and dune erosion processes

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ABSTRACT

This paper is focussed on the derivation of a set of general scaling laws valid for both beach and dune erosion volumes based on scaling law analysis, existing and new experimental results. This latter experiments concern beach profile changes in three different laboratory flumes using identical wave conditions based on Froude scaling. The experiments with planar sloping beaches have been done at three scales: large-scale Hannover wave flume experiment (beach slope of 1 to 15), medium scale Barcelona wave flume experiment (beach slope of 1 to 15) and small-scale Delft wave flume experiments (beach slopes of 1 to 10, 15 and 20) using an identical wave train of irregular waves (single topped spectrum).

The available data sets have been used to derive a general set of scaling laws which is valid for both beach and dune erosion under storm conditions. The morphological time scale depends on the depth scale and the length scale, and only indirectly on the sediment size scale. The analysis of the scaling laws shows that laboratory scale models can be operated at distorted scales, if necessary. Finally, two applications of the scaling laws are presented.

1. Introduction

Physical scale models of sandy (quartz) materials have been used frequently to study coastal engineering problems. Scaling laws for coastal movable bed models are well established (Le Méhauté, 1970; Noda, 1972; Kamphuis, 1972, 1982; Hughes, 1983, 1993), but the errors due to scale effects are less well understood.

The basic philosophy for movable-bed models can be formulated as ensuring that the relative magnitudes of all dominant processes are the same in model and prototype. Preferably, the scale model should be validated using field (prototype) data, but often this is not feasible and large-scale model results are used as prototype data. For coastal scale models the most relevant requirement is to attain similarity of the cross-shore equilibrium bed profiles between prototype and model, particularly in the surf zone (Hughes and Fowler, 1990). In practice, most relevant is the proper representation of the beach and dune erosion volumes. This means that the dimensionless parameters describing the equilibrium erosion volumes should be the same in model and prototype. The most dominant mode of transport being either bed load or suspended load (depending on wave conditions and bed material) should be represented correctly. Both undistorted models and distorted models have been used in scale modelling. Ensuring similarity in an undistorted model is less complicated than in a distorted model. Dean (1985) reasoned that, in an undistorted model with a sand bed, the fall trajectory of a suspended particle must be geometrically similar to the equivalent prototype trajectory and fall with a time proportional to the prototype fall time. This can be accomplished by ensuring similarity of the fall velocity parameter between the prototype and the model, which is only feasible in an undistorted model. Distorted models can be used when the reproduction of the bulk erosion volume (dune erosion volume; Vellinga, 1986) is most important, while the precise reproduction of the equilibrium profile in the entire surf zone is less important.

This paper is focussed on the derivation of a set of general scaling laws valid for both beach and dune erosion volumes based on scaling law analysis, existing and new (SANDS EU-Project) experimental results. The old experiments of Vellinga (1986) have been re-analyzed. The SANDS EU-Project (2005–2009) was focussed on the comparison of a series of laboratory scale experiments using identical wave conditions based on Froude scaling. The experiments with planar sloping beaches have been done at three scales: large-scale Hannover wave flume experiment (beach slope of 1 to 15), medium scale Barcelona wave flume experiment (beach slope of 1 to 15) and small-scale Delft wave flume experiments (beach slopes of 1 to 10, 15 and 20) using an identical wave train of irregular waves (single topped spectrum).

2. Definitions and existing scaling laws for coastal processes

2.1. Definitions

In physical scale modelling the basic parameters (wave height, length, orbital velocity, etc.) are generally much smaller than the

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corresponding values in nature. The ratio of the value in nature (prototype) and in the laboratory model is generally expressed by the scale parameter $n = p_p / p_m$ with p_p = parameter value in prototype and p_m = parameter value in laboratory model. Thus, $n > 1$.

Correct representation of the physical processes in nature requires that the dimensionless numbers (Froude number, Reynolds number, etc.) characterizing these processes are the same in nature and in the laboratory model. Examples of these numbers are: the Froude number (subcritical or supercritical flow), the Reynolds number (laminar or turbulent flow), the surf similarity parameter (type of breaking), the Suspension parameter (bed load or suspended load transport), the Shields parameter (intensity of sediment transport and type of bed forms). Often, it is sufficient for correct scale modelling that these dimensionless numbers are in a certain range rather than imposing a fixed value. For example, it is often sufficient that the flow is turbulent in the laboratory model which is satisfied if the Reynolds number is larger than about 1000.

2.2. Existing scaling laws

Correct representation of the wave dynamics requires (see Noda, 1972; Kamphuis, 1972; Vellinga, 1986):

$$n_u = n_T = (n_L)^{0.5} = (n_H)^{0.5} = (n_h)^{0.5} \tag{2.1}$$

with: u = orbital velocity, T = wave period, H = wave height, L = wave length, and h = water depth.

Generally, Eq. (2.1) is known as Froude scaling.

Noda (1972) has performed various beach erosion experiments in scale models (distorted and undistorted) with relatively low steepness, regular waves focussing on relatively coarse sand (>0.5 mm; initial slope of 1 to 25). Based on analysis of quasi-equilibrium bed profiles, he proposed the following scaling laws:

$$n_{d50} = (n_h)^{0.55} \tag{2.2a}$$

$$n_l / n_h = (n_h)^{0.32} \tag{2.2b}$$

with: n_l = horizontal length scale, n_h = depth scale, and n_{d50} = sediment size scale.

Ito and Tsuchiya (1984, 1986, 1988) and Ito et al. (1995) have done detailed studies of beach erosion profiles in quasi-equilibrium conditions under low and high waves. They have used small-scale and large-scale physical models (undistorted) with regular waves and initial slopes of 1 to 15 and 1 to 30. Similitude between bed profiles of different scales is defined to exist when the difference is less than twice the experimental error (based on repeated tests). The low-wave cases show onshore transport with the formation of a swash bar at the upper end of the beach slope (1 to 30), while the high-wave cases show offshore transport with the formation of a breaker bar at the lower end of the beach slope (1 to 15). The prototype bed profiles are obtained from the results measured in large-scale wave flumes (offshore depth of 1 to 4.3 m; wave periods of 3 to 11 s).

The scaling laws derived from these undistorted scale model (regular waves) series are:

$$n_{d50} = (n_h)^{0.83} \qquad \text{for } n_h < 2.2 \tag{2.3a}$$

$$n_{d50} = 1.7(n_h)^{0.2} \qquad \text{for } n_h \geq 2.2 \tag{2.3b}$$

$$n_{TM} = (n_h)^{0.5} \tag{2.3c}$$

with: n_{TM} = morphological time scale.

These scaling laws were applied to a storm-induced beach erosion event (14–18 March 1981) on the Ogata coast facing the Pacific Ocean (Japan). The significant wave height increased from 0.5 m to about 4 m in about one day, remained constant for the following day and decreased again after that. The storm-induced set-up was about 0.3 m. The beach sediments varied in the range of 0.2 mm in the offshore zone to about 0.4 mm at the beach. The depth scale was set to $n_h = 50$, the grain size scale was set to $n_{d50} = 3.7$ based on Eq. (2.3b). Two model sands were used to represent the beach material variation in the prototype. The wave height variation during the storm event was represented by schematizing it into three regular wave cases, each with constant but different wave height and period.

The beach profile changes observed in the prototype are reproduced very well when the mean wave height was used as the representative wave height in the prototype and slightly less good when the significant wave height was used. Similar conclusions are given by Ito et al. (1995).

Wang et al. (1990) have proposed:

$$n_l / n_h = \left[(n_h)(n_{s-1})^{-2}(n_{ws})^{-2}\right]^{\alpha} \tag{2.4}$$

with: n_{ws} = fall velocity scale, n_{s-1} = relative density scale.

Method A $\alpha = 0.5$ for $n_T = n_{TM} = (n_h)^{0.5}(n_l / n_h)$
Method B $\alpha = 0.25$ for $n_T = (n_h)^{0.5}(n_l / n_h)$ and $n_{TM} = (n_h)^{0.5}$

They have found that Method B yields better agreement in terms of the morphological time scale and that the similarity of the wave breaking process (surf similarity parameter) and the fall velocity parameter are the most important considerations for correct modelling of beach processes.

Hughes (1993) has proposed a distorted model time scale which is similar to the preservation of the surf similarity parameter (see also Eq. (3.2)): $n_T = n_{TM} = (n_h)^{0.5}(n_l / n_h)$.

Hughes and Fowler (1990) have performed small-scale model experiments (undistorted) aimed at reproducing large-scale experiments in the GWK (Grosser Wellen Kanal, Hannover); the latter being defined as the prototype. The prototype sediment is $d_{50} = 0.33$ mm and $w_s = 0.0447$ m/s. The depth at the toe of the initial profile is 5 m for the prototype and 0.67 m for the scale model. The wave period is 6 s for the prototype and 2.2 s for the scale model. Using an undistorted model, the applied scaling laws are:

$$n_{ws} = (n_h)^{0.5} \tag{2.5a}$$

$$n_{TM} = (n_h)^{0.5} \tag{2.5b}$$

Given a depth scale of $n_h = 7.5$, the fall velocity scale is $n_{ws} = 2.73$ (scale model sand of 0.13 mm, $w_s = 0.0164$ m/s). In the prototype experiments, sand with a median diameter of 0.33 mm was placed under a slope of 1 to 4 in front of a concrete structure with a slope of 1 to 4. Both regular and irregular (Jonswap) wave tests were done. In the case of regular waves (H = 1.5 m for prototype and 0.2 m for scale model) the model erosion at the upper end of the beach was slightly (10%) under-estimated. Almost perfect agreement was obtained by increasing the wave height by about 10%. Comparing test results of irregular waves ($H_{1/3} = 1.5$ and 0.2 m; Tp = 6 and 2.2 s for prototype and scale model) in the prototype and in the scale model, the agreement was found to be very good, see Fig. 2.1.

Comparable profile development can be achieved between regular waves in the scale model and irregular waves in the prototype when the value of significant wave height is used as the regular wave height in the scale model. Profile development is found to be approximately twice as fast in the scale model with regular waves. Hence, only half the number of waves is required to obtain the same quasi-equilibrium bed profile (at corresponding times). In all tests the (offshore) slope at the toe of the beach was much steeper in the prototype (0.33 mm) than in the scale model (0.15 mm), see Fig. 2.1. Probably, there was

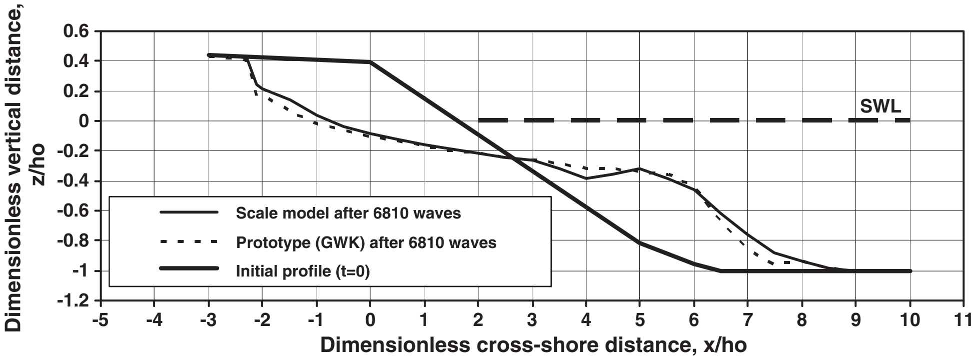

Fig. 2.1. Comparison of cross-shore bed profiles of scale model and prototype (irregular waves). Hughes and Fowler, 1990.

relatively large onshore transport at this location (shoaling waves) in the prototype (GWK, Hannover) causing a steeper slope.

Dette and Uliczska (1987a,b) have compared beach profiles of the GWK (Hannover) with similar tests performed at scale of 1 to 10 ($n_h = 10$). The scale model sand (0.33 mm) was similar to that used in the GWK. The best agreement of the beach profiles in the surf zone was found when the scaling laws of Vellinga (see Eqs. (2.6a), (2.6b), (2.6c), and (2.6d)) were used. In the offshore zone where bed-load transport is dominant the scaled-up model results were substantially too shallow (less steep profiles in scale model).

Based on scale model results, Vellinga (1986) has developed a simple rule for dune erosion by storms (DUROS-model). The shape of the erosion profile is defined by a parabolic function which is independent of the initial profile; the most seaward position of the erosion profile is determined by the offshore wave height and the beach grain size. The dune erosion volume above storm surge level is determined by shifting this profile horizontally until the erosion volume is equal to the deposition volume (per unit width).

According to Vellinga (1986), the scaling laws for dune erosion read as:

$$(n_l \,/\, n_h) = (n_h)^{0.28} (n_w)^{-0.56} \tag{2.6a}$$

$$n_A = n_l n_h \tag{2.6b}$$

$$n_{TM} = (n_h)^{0.5} \tag{2.6c}$$

Combining Eqs. (2.6c) and (2.6a) yields:

$$n_{TM} = (n_l \,/\, n_h)(n_h)^{0.22} (n_w)^{0.56} \tag{2.6d}$$

with: n_w = fall velocity scale, n_{TM} = morphological time scale, and n_A = erosion volume scale.

The practical ranges are: $n_l/n_h = 1$ to 2; $n_h = 1$ to 50; $n_{d50} = 1$ to 5 and $n_{ws} = 1$ to 5.

Ranieri (2007) has made the most recent contribution to the scaling laws of beach profiles. Test results of the GWK (Hannover) have been defined as the prototype results. A series of undistorted scale models (n_h in the range of 10 to 45; Froude scaling) has been executed to determine the scale effects. He has proposed to correct the scale errors based on a distortion coefficient (α_{surf}) to be applied only to the surf zone between the bar crest and the water line. The erosion volume in the prototype (A_p) can be computed by:

$$A_p = \alpha_{surf} n^2 A_m \tag{2.7a}$$

with: A_m = erosion volume of scale model; $n = n_h = n_l$ = length scale of undistorted model and $\alpha_{surf} = n_{l,s}/n_h$ = correction factor in the range of 1 to 2.5 acting in surf zone only (approximately between bar crest and water line).

The α_{surf} is found to depend on:

$$\alpha_{surf} = -1.42 + 1.12\, ln\left(n_\phi / n_k\right) \quad \text{for } n_\phi > 5 \tag{2.7b}$$

$$\alpha_{surf} = 1 \quad \text{for } n_\phi \leq 5 \tag{2.7c}$$

with: $n_\phi = n/(n_{s-1}\ n_{d50})$ and $n_k = (\Phi_{95} - \Phi_5)/(2.44(\Phi_{75} - \Phi_{25}))$, with: $\Phi_i = {}^2\log(d_i)$ and d_i = particle size.

The scale models used by Ranieri have been initially set up as undistorted models using Froude scaling. The correction coefficient acting on the horizontal scale has been obtained by requiring optimum agreement between the quasi-equilibrium profiles of the scale model and the prototype. Correction has been applied only to the surf zone between approximately the bar crest and the water line. This method allows to design very small undistorted models and then to correct the results by using the α_{surf}-coefficient.

2.3. Scaling problems

The design of movable-bed scale models involves various problems related to the selection of the appropriate scales, as follows:

- the scale model depth should not be so small that the flow (unidirectional and/or oscillatory) is no longer in the fully turbulent regime or that surface tension effects become dominant;
- the scale model shear stress should be sufficiently large that it is significantly beyond the critical bed-shear stress for initiation of motion; this requirement often implies a scale distortion in bed roughness (ripples in model; sheet flow in prototype);
- the scale model sediment should not be so small that cohesive properties become important;
- the scale model distortions should not be so large that the type of sediment transport (either dominant bed load or suspended load) in the spatial domain of interest (either offshore or inshore) is incorrect; as scale distortion is controversial (contradictory views) it should be kept to a minimum value.

3. Analysis of scaling laws for coastal processes

The dominant processes in the nearshore coastal zone are: wave shoaling, wave breaking, surf beat and wave run-up, generation of cross-shore return currents (undertow), wave-induced sediment mobility, suspension and suspended transport and dune face slumping.

3.1. Wave breaking

The type of wave breaking is generally characterized by the surf similarity parameter, expressed as: $\xi = \tan(\beta)/(H/L)^{0.5}$, with $\tan(\beta)$ = local bed slope($=\Delta h/\Delta l$). Spilling breaking occurs for $\xi<0.4$, plunging waves for $0.4<\xi<2.3$ and surging waves for >2.3

Correct representation of wave breaking requires:

$$n_\xi = 1 \text{ or } n_h / n_l = (n_H / n_L)^{0.5} \quad (3.1)$$

Using: $n_H = n_h$ and $n_L = (n_T)^2$ from Eq. (2.1) it follows that:

$$n_T = (n_h)^{0.5}(n_l / n_h) \quad (3.2)$$

Eq. (3.2) is known as the distorted Froude scaling. The ratio n_l/n_h expresses the distortion scale. Generally, it is required that $n_l > n_h$ to fit the relatively long cross-shore profile in the laboratory model. Eq. (3.2) shows that correct representation of the wave dynamics including wave breaking can only be represented in a non-distorted model ($n_l = n_h$). In a distorted model ($n_l/n_h > 1$) with a steeper bed slope, the waves should be shorter (at the same wave height) to obtain the same surf similarity parameter. In practice this latter parameter is not so important as long as the waves are breaking and are in a certain breaking range (either spilling or plunging). More intense spilling of more intense plunging is not so important. Hence, in most cases the waves are scaled using the Froude scaling (Eq. (2.1)) rather than the distorted Froude scaling (Eq. (3.2)).

3.2. Undertow

The erosion processes at the beach during wave attack are mainly governed by the transport capacity of the undertow (wave-driven flow). Correct representation of the undertow velocity requires that: $n_{ur}/n_{cw} = 1$, with u_r = undertow velocity and c_w = wave celerity. Based on linear wave theory in shallow water (mass flux theory), the undertow is proportional to: $u_r \approx gH^2/(c_w h)$.

Thus, $u_r/c_w \approx gH^2/(c_w^2 h)$.

From $n_{ur}/n_{cw} = 1$, it follows that: $(n_T)^2(n_H)^2 = n_h(n_L)^2$, which results in:

$$n_T = (n_h)^{0.5} \quad (3.3)$$

Eq. (3.3) is similar to Eq. (2.1); Froude scaling.

3.3. Wave runup along dune face

The wave runup along the dune face can be represented by:

$$RU = \alpha g^{0.5} T H^{0.5} \tan(\beta) \quad (3.4)$$

with: RU = wave runup (vertical level above still water level) exceeded by 2% of the waves, α = coefficient ($\cong 0.7$), $\tan(\beta)$ = local bed slope.

This yields:

$$n_{RU} = n_T(n_H)^{0.5}(n_h / n_l) = n_h(n_h / n_l) \quad (3.5a)$$

The wave runup scale is only equal to the vertical depth scale in the case of a non-distorted model ($n_l/n_h = 1$).

When a distorted model (with $n_l > n_h$) is used, the runup scale will be overestimated in the model, which may result in overestimation of the erosion. Assuming that $n_l/n_h = (n_h)^\alpha$, it follows that:

$$n_{RU} = (n_h)^{1-\alpha} \quad (3.5b)$$

This means that the runup scale will a factor $(n_h)^{1-\alpha}$ too large in a distorted model. For example, $\alpha = 0.25$ and $n_h = 6$ and $n_l/n_h = (6)^{0.25} = 1.55$, then $n_{RU} = (6)^{0.75} = 3.8$. Ideally, the runup scale should actually be equal to 6. The runup scale is thus a factor $6/3.8 = 1.55$ ($=6^{0.25}$) too large in a distorted model.

3.4. Suspension parameter

Dune and beach erosion in breaking wave conditions (surf zone) over a sandy bed is largely controlled by suspension processes, characterized by the dimensionless number $H/(w_s T)$, which should be the same in nature in the model. The dimensionless fall velocity parameter $H/(w_s T)$ known as the Dean-number, was popularized by Dean (1973, 1985). This parameter represents the time taken by a sediment particle to travel a vertical distance equal to the wave height. Suspended load dominates for $H/(w_s T) \ll 1$. Similar results have been obtained by Le Méhauté (1970) based on the similarity of ratio U/w_s with U = horizontal peak orbital velocity and w_s = fall velocity.

Thus:

$$n_H = n_{ws} n_T \text{ or } n_T = n_h / n_{ws}. \quad (3.6)$$

Combining Eqs. (3.6) and (3.2), yields:

$$n_l / n_h = (n_h)^{0.5} / n_{ws} \quad (3.7a)$$

or,

$$n_{ws} = (n_h)^{0.5}(n_l / n_h)^{-1} \quad (3.7b)$$

Using $n_{ws} = n_{d50}$ for sediments of 0.1 to 0.5 mm, it follows that:

$$n_{d50} = (n_h)^{0.5}(n_l / n_h)^{-1} \quad (3.7c)$$

Eqs. (3.7a), (3.7b) gives a scale relationship for distorted models. The value of n_l can be found after selecting n_h and n_{ws} or n_{d50}.

By using Eqs. (3.7a), (3.7b), (3.7c) the modelling of wave breaking and suspension processes is fairly correct, but no information is available on the morphological time scale involved. This latter parameter can only be determined by considering the transport rates at the toe of the beach and/or dune in comparison to the amount of sediment eroded from the beach and/or dune face.

Similarity of sedimentation processes in quiescent areas (trough zone landward of the bar crest) can be obtained by assuming that the horizontal suspended transport gradient is approximately equal to the vertical deposition flux at the bed:

$$d(huc) / dx = w_s c \quad (3.7d)$$

Eq. (3.7d) applies to both the prototype and the scale model. Similarity is preserved, if:

$$n_{ws} = n_u(n_l / n_h)^{-1} = (n_h)^{0.5}(n_l / n_h)^{-1} \quad (3.7e)$$

Eq. (3.7e) is the same as Eq. (3.7b).

3.5. Mobility parameter

Correct representation of the bed forms and bed-load transport in nature and model requires that the sediment mobility parameter $f(U_{max})^2/[(s-1)gd_{50}]$ is equal in both cases (f = friction factor). Thus:

$$n_{mob} = 1 \text{or} (n_u)^2 = n_{s-1} n_{d50} (n_f)^{-1} \quad (3.8a)$$

Based on linear wave theory: $U_{max}=\pi HT^{-1}\sinh(kh)$ with $k=2\pi/L$. In shallow water: $\sinh(kh)=kh$. Thus:

$$U_{max}=2\pi^2 Hh\ T^{-1}\ L^{-1} \tag{3.8b}$$

Using $n_H=n_L=n_h$ and Eq. (3.2): $n_T=(n_h)^{0.5}(n_l/n_h)$, it follows that:

$$n_U=(n_h)^{0.5}(n_l/n_h) \tag{3.8c}$$

Combining Eqs. (3.8a) and (3.8c), yields:

$$(n_u)^2=n_h(n_l/n_h)^2\, n_{d50}(n_f)^{-1} \text{ or,}$$

$$n_{d50}=n_h n_f(n_{s-1})^{-1}(n_l/n_h)^{-2} \tag{3.8d}$$

Using sand in a non-distorted model, if follows that:

$$n_{d50}=n_h n_f \tag{3.8e}$$

The friction factor scale generally is much smaller than 1 ($n_f\ll 1$) for storm conditions (flat bed in prototype and ripples in model; $f_p\cong 0.01$ and $f_m\cong 0.05$ or $n_f\cong 0.2$). This behaviour is favourable for modelling purposes as it leads to coarser model sediment. For example: $n_h=25$, $n_f=0.2$ and thus $n_{d50}=5$. Preferably, regular waves should not be used in a scale model because the ripples may induce offshore-directed sand transport against the wave direction (in the case of non-breaking waves).

As regards fully developed sand transport, it is essential that the dimensionless excess shear stress or excess velocity ($U_{max}-U_{cr}$) is reproduced correctly (U_{cr} = critical orbital velocity for initiation of motion). This requires that the mobility number $f(U_{max}-U_{cr})^2/[(s-1)gd_{50}]$ is equal in both model and prototype or:

$$(n_{u-ucr})^2=n_{s-1}n_{d50}(n_f)^{-1} \tag{3.8f}$$

Scale effects will be relatively small as long as $U\gg U_{cr}$ in the scale model and relatively large for $U\cong U_{cr}$ or $U<U_{cr}$ in the scale model. In the latter case no sediment motion will occur in the model. This may occur when relatively coarse sand is used in the scale model. These scale errors generally are largest outside the surf zone (offshore slope of outer bar). Scale conditions close to initiation of motion should be avoided as much as possible.

The mobility parameter is most important in shoaling waves with dominant sand transport close to the bed. This parameter is less important in the surf zone with strongly breaking waves.

3.6. Suspended transport at toe of beach or dune and morphological time scale

The suspended transport rate in the surf zone is given by: $q_s=huC$ with q_s = transport rate in m^2/s, h = water depth, u = undertow velocity and C = concentration. It is assumed that the concentration is fairly constant over the depth in the inner surf and swash zone with strongly breaking waves (Van Rijn, 1993, 2006).

The suspended sediment concentration in the lower half of the water column is assumed to be proportional to (Van Rijn, 1993, 2006):

$$C\approx\frac{(U)^a(SL)^b}{(T)^c(d_{50})^d(s-1)^e} \tag{3.9a}$$

with: U = peak orbital velocity, SL = bed slope, T = wave period, d_{50} = median sediment size of bed material, and s = relative density ($=\rho_s/\rho_w$).

From basic sediment research in laboratory flumes it is known that approximately (Van Rijn, 1993, 2006):

$$C\approx U^3, C\approx 1/(T)^{1\text{to}2}, C\approx 1/(d_{50})^{1\text{ to }2} \text{ and } C\approx 1/(s-1)$$

The effects of bed slope on the depth-averaged concentration is less well known, but it is herein assumed that the concentration increases with increasing bed slope (concentration $\approx SL^{0.5\text{ to }2}$). Thus, a = 2 to 3, b = 0.5 to 2, c = 1 to 2, d = 1 to 2 and e = 1.

Using $n_U=(n_h)^{0.5}$, $n_T=(n_h)^{0.5}$ and $n_{SL}=n_h/n_l$, the suspended sand concentration scale n_c can be represented as:

$$n_c=(n_h)^{0.5a-0.5c}(n_{d50})^{-d}(n_{s-1})^{-e}(n_l/n_h)^{-b} \tag{3.9b}$$

The suspended transport scale ($n_{qs}=n_h n_u n_c$) can be represented as:

$$n_{qs}=(n_h)^{1.5+0.5a-0.5c}(n_{d50})^{-d}(n_{s-1})^{-e}(n_l/n_h)^{-b} \tag{3.9c}$$

The transport rate at the seaward toe of the beach or dune is also equal to:

$$q_s=A_e/T_M \tag{3.10}$$

with: A_e = erosion volume per unit width and T_M = time scale to erode the beach or dune face.

The scale relationship related to Eq. (3.10) is:

$$n_{qs}=(n_h)(n_l)/n_{TM}=(n_l/n_h)(n_h)^2/(n_{TM}) \tag{3.11}$$

From Eqs. (3.11) and (3.9c), it follows that:

$$n_{TM}=(n_l/n_h)^{b+1}(n_{d50})^d(n_{s-1})^e(n_h)^{0.5-0.5a+0.5c} \tag{3.12a}$$

The morphological time scale can be expressed as:

$$n_{TM}=n_h^{\alpha} \tag{3.12b}$$

Using $\alpha=0.5$, the morphological time scale is equal to wave period time scale: $n_{TM}=n_T=n_h^{0.5}$.

Using Eqs. (3.12b) and (3.12a), it follows that:

$$(n_l/n_h)^{b+1}=(n_{d50})^{-d}(n_{s-1})^{-e}(n_h)^{\alpha-0.5+0.5a-0.5c} \tag{3.13}$$

For α in the range 0.5 to 1.0, it follows that:

$$(n_l/n_h)=(n_{d50})^{-0.5\text{to}-1}(n_{s-1})^{-0.5}(n_h)^{0.25\text{ to }0.50} \tag{3.14a}$$

Eq. (3.9a) can also be formulated in terms of the fall velocity of the sediments (w_s) instead of the median sediment diameter (d_{50}). Using $w_s\approx d_{50}$ for fine sand and $\alpha=0.5$ to 1.0, this yields:

$$(n_l/n_h)=(n_{ws})^{-0.5\text{ to }-1}(n_{s-1})^{-0.5}(n_h)^{0.25\text{ to }0.50} \tag{3.14b}$$

Using sand with density of 2650 kg/m^3, it follows that: $n_{s-1}=1$. If the same sand is used in model and prototype (or $n_{d50}=1$), it follows that:

$$(n_l/n_h)=(n_h)^{0.25\text{ to }0.50} \tag{3.14c}$$

Based on analysis of many scale model results on dune erosion (sand with $n_{d50}=1$; same sand in model and prototype), Van de Graaff (1977) has proposed:

$$(n_l/n_h)=(n_h)^{0.28} \tag{3.14d}$$

The concentration scale is proposed to be based on Eqs. (3.9a), (3.9b):

$$n_c = (n_h)^{0.5}(n_{d50})^{-1 \text{ to } 2}(n_{s-1})^{-1}(n_l/n_h)^{-1} \quad (3.15)$$

Laboratory model data are required to determine the proper values of the exponents.

4. Analysis of scale model results and errors for beach and dune erosion

4.1. New experimental results of beach erosion (SANDS experiments)

4.1.1. Sands beach erosion experiments

The SANDS EU-Project (2005–2009) is focussed on the comparison of a series of laboratory scale experiments using identical wave conditions based on Froude scaling. The experiments have been done at three scales: large-scale Hannover wave flume experiment (beach slope of 1 to 15), medium scale Barcelona wave flume experiment (beach slope of 1 to 15) and small-scale Delft wave flume experiments (beach slopes of 1 to 10, 15 and 20) using an identical wave train of irregular waves (single topped). The Hannover wave flume is defined to be the prototype. So, the tests in the other wave flumes can be seen as model tests at scale n. The experimental set-up consists of a horizontal bed followed by a plane sloping beach. The main sediment and hydrodynamic parameters are presented in Table 4.1A.

4.1.2. Beach erosion profiles and volumes

Figs. 4.1 to 4.3 show the dimensionless beach profiles (initial slope of 1 to 15) at various times in the three wave flumes. The horizontal and vertical scales are made dimensionless using the incoming wave height ($H_{m,o}$) of each experiment. The vertical scale is defined with respect to the still water line (SWL). The origin of the horizontal scale is defined at $x/H_{m,o}=200$ from the toe point of the initial plane beach

Comparison of these profiles shows that the profiles in the erosion zone around the still water line (SWL) are similar, although the erosion profiles in the Hannover wave flume are somewhat smoother and less triangular as in the Delft wave flume. The dimensionless length of the erosion zone is roughly $L_e/H_{m,o}=25$ for the Hannover and Barcelona experiments and about $L_e/H_{m,o}=35$ for the Delft experiment.

The dimensionless length of the deposition zone is about $L_{h,d}/H_{m,o}=25$ for the Hannover and Barcelona experiments and is about $L_{h,d}/H_{m,o}=45$ for the Delft experiment with relatively fine sand. The maximum dimensionless vertical erosion depth is about $L_{v,e}/H_{m,o}=0.5$ to 0.75 for the Hannover and Barcelona experiments and about 1.25 for the Delft experiment. The maximum vertical dimensionless deposition height is approximately $L_{v,d}/H_{m,o}=1$ for all three experiments. The deposition profiles have a typical bar shape in the Hannover wave flume where the coarsest sand has been used (0.27 mm). The profiles in the deposition zone below SWL are most smooth in the Delft wave flume where the finest sand has been used (0.13 mm). Overall, the results of the Hannover and Barcelona experiments are quite similar whereas the results of the Delft experiment are somewhat deviating due to the use of relatively fine sand.

4.1.3. Analysis of erosion volumes

Using Eq. (3.12a), the morphological time scale can be expressed as:

$$n_{TM} = (n_l/n_h)^p(n_{d50})^q(n_{s-1})^r(n_h)^s \quad (4.1)$$

The density effect is assumed to be represented by r = 1.

Based on the measured bed profiles, the erosion volumes (A_e in m^3/m) with respect to the initial profile have been computed and made dimensionless by dividing with the offshore wave height to the power of 2 ($H^2_{m,o}$). The results are plotted as a function of time, see Fig. 4.4. The erosion in the small-scale Delft experiment proceeds much faster than that in the Hannover experiment. Comparing the curves of Hannover-Delft ($n_h=6.2$, $n_{d50}=2.5$) and Hannover–Barcelona ($n_h=1.85$, $n_{d50}=1.3$), the time scale factor to collapse the curves into one plot are 2 and 1.2. These results can be represented by: $n_{TM}=(n_{d50})^0(n_h)^{0.4}$. The sediment scale has almost no effect on the results.

The Delft experiment ($n_{d50}=1$, $n_h=1$, and $n_{s-1}=1$) with beach slopes of 1 to 10 (thus $n_l/n_h=2$) and 1 to 15 (thus $n_l/n_h=1.33$) can be seen as distorted tests compared to the test with beach slope of 1 to 20. Fig. 4.5 shows the dimensionless beach erosion volumes (made dimensionless by $H^2_{m,o}$) as function of time for the three tests. These plots collapse into a single function using the time scale law: $n_{TM}=(n_l/n_h)^{1.4}$

Combining the results of all three flumes, the morphological time scaling law now is:

$$n_{TM} = (n_{s-1})(n_l/n_h)^{1.4}(n_h)^{0.4} \quad (4.2)$$

To ensure similarity of suspension processes in the case of fine sediment beaches and dunes (0.1 to 0.5 mm), another expression is required relating the distortion scale, the depth scale, the sediment size scale and the sediment density scale. This relationship can be derived from dune erosion data sets with distorted model scales (many data available; see Section 4.2).

Table 4.1A
Data of SANDS model scale experiments in wave flumes of Hannover, Barcelona and Delft.

Parameter		Hannover wave flume	Barcelona wave flume	Delft wave flume
Water depth above horizontal flume bottom (to SWL)		4.2 m	2.47 m	0.7 m
Water depth above horizontal sand bed (to SWL)		3.2 m	1.77 m	0.5 m
Beach slope		1 to 15	1 to 15	1 to 15, 10 and 20
Height of sand dune above SWL		1.5 m	0.7 m	0.4 m
Waves (erosive test)	$H_{m,o}$	0.97 m	0.53 m	0.167 m
	T_p	5.6 s	4.14 s	2.33 s
Waves (accretive test)	$H_{m,o}$	0.60 m	0.36 m	0.1 m
	T_p	7.7 s	5.1 s	3.06 s
Sand	d_{10}	0.14	0.16 mm	0.10 mm
	d_{50}	0.27 mm	0.25 mm	0.13 mm
	d_{90}	0.51	0.37 mm	0.17 mm
Depth scale n_h		1	1.8	5.8 to 6.4
Sediment scale n_{d50}		1	1.08	2.08
Distortion scale n_l/n_h		1	1	1, 1.5 and 0.5

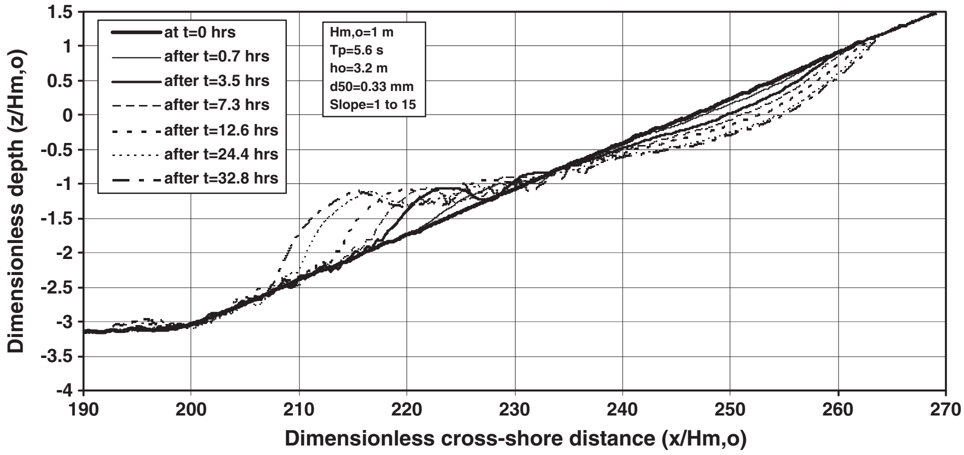

Fig. 4.1. Measured dimensionless beach profiles as function of time in Hannover wave flume.

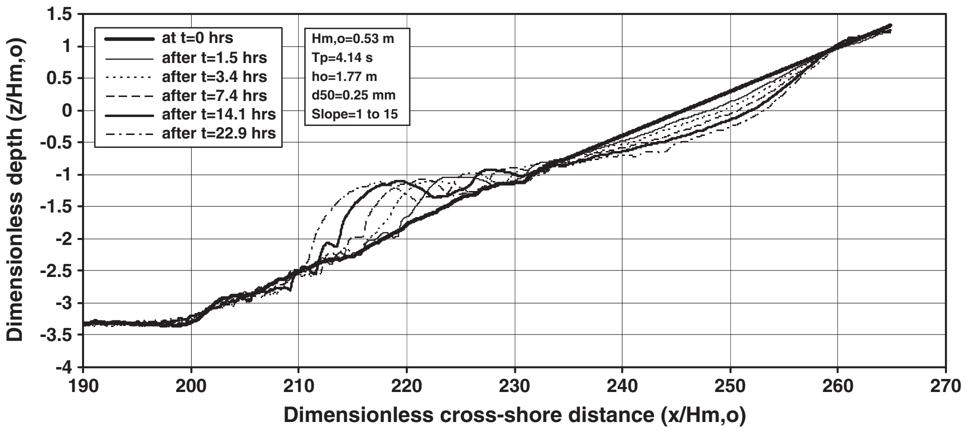

Fig. 4.2. Measured dimensionless beach profiles as function of time in Barcelona wave flume.

4.2. Available experimental results of dune erosion

4.2.1. Dune erosion experiments

The SANDS data basically represent beach erosion under moderate storm events. To evaluate the predictability of the scaling law Eq. (4.2), various dune erosion experiments under extreme storm events have also been also analyzed.

Experimental data of scale tests on dune erosion have been collected by Vellinga (1986) and Delft Hydraulics (2004, 2006a,b, 2007). The basic experimental data are given in Tables 4.1A, 4.1B, 4.2A, 4.2B, 4.3A, and 4.3B. Tables 4.2A and 4.2B represent a selection of the Vellinga tests eliminating tests with inconsistent results. The experimental data typically represent beach and dune erosion conditions along the Dutch North Sea coast during a very severe storm (design storm), which is herein defined as the Reference Case, see Table 4.1B. The median sediment diameter along the Dutch coast is assumed to be 0.225 mm. The high storm surge level (SSL) of 5 m above MSL is assumed to be constant over a duration of 5 h during the peak of the storm. This equivalent duration of 5 h yields approximately the same overall dune erosion volume as a complete storm cycle with growing and waning

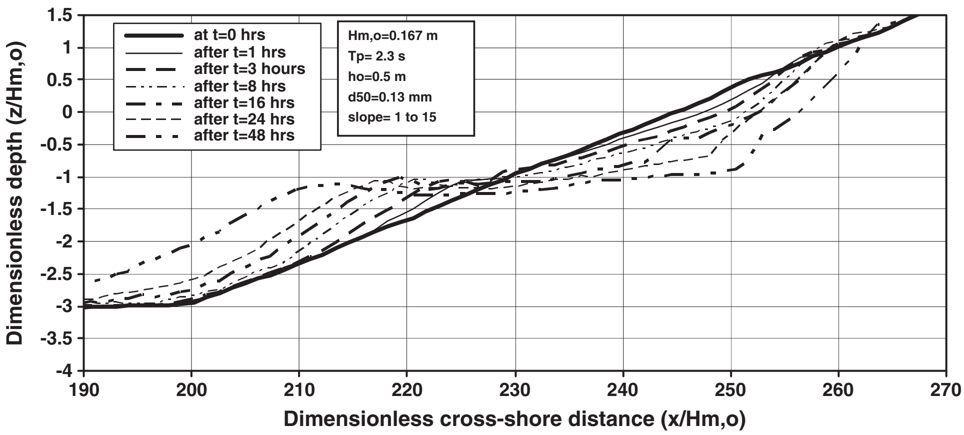

Fig. 4.3. Measured dimensionless beach profiles as function of time in Delft wave flume.

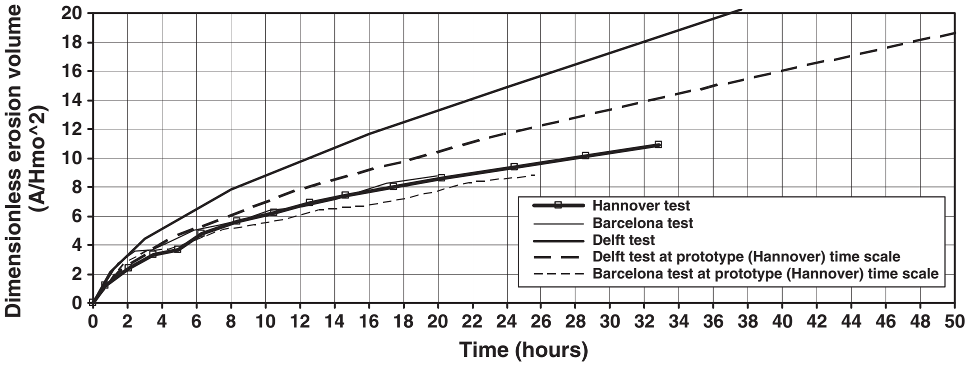

Fig. 4.4. Measured dimensionless beach erosion volume as function of time; Hannover, Barcelona and Delft experiments.

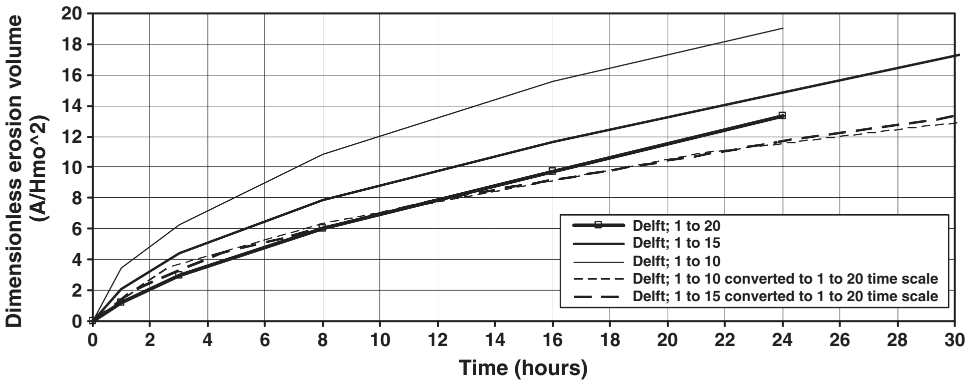

Fig. 4.5. Measured dimensionless beach erosion volume as function of time; Delft experiments.

phases (Vellinga, 1986). The offshore significant wave height is assumed to have a constant value of $H_{s,o} = 7.6$ m and the peak wave period is $T_p = 12$ s.

The offshore wave height in the Deltaflume is about 1.5 m. The measured offshore wave height during the large-scale tests in the Deltaflume of Vellinga (1986) appeared to be somewhat larger (1.8 m) than originally intended (1.5 m), see page 103 and 133 of Steetzel (1993). Given the applied depth scale of $n_h = 5$, this results in a deep water wave height of $H_{s,o} = 9$ m. This latter value was used in later tests.

4.2.2. Analysis of dune erosion volumes

The median sediment diameter of the scale test series of Vellinga (1986) was varied in the range of 0.095 to 0.225 mm; thus: $n_{d50} = 2.4$ to 1. The vertical scale is in the range between $n_h = 84$ and $n_h = 5$. Large scale tests with $n_h = 5$ and 6 were performed in the Deltaflume (length of 233 m, depth of 7 m, width of 5 m) of Delft Hydraulics (2006a,b). It is noted that the data of some tests with very small water depths and relatively coarse sand have been neglected (in this study) due to the presence of scale effects. Some of these tests with very small depths show decreasing erosion values in time probably due to onshore transport effects. The combination of very small waves and coarse sediment may easily lead to onshore bed load transport rather than offshore suspended load transport (as present in prototype conditions). The combination of very small waves and fine sediment may lead to hydraulically smooth flow conditions. In the latter case the fine sediment particles are buried in the laminar sublayer leading to much larger critical shear stresses (left limb of the Shields curve) and hence reduction of the transport rate.

Ten representative tests were selected to plot the erosion volumes as a function of time (on prototype scale in m^3/m; $n_h = 1$), using Eq. (4.2). The results are shown in Fig. 4.6. The results with small depth scales ($n_h = 26$ to 84) are relative close together, but the large-

Table 4.1B
Parameters of Dutch coastal profile; Reference Case.

Parameter	Prototype conditions used by Vellinga (1986)	Prototype conditions used by Delft Hydraulics (2004, 2006a,b)
Offshore wave height (m)	7.6 (Pierson and Moskowitz spectrum)	9.0 (Pierson and Moskowitz spectrum)
Offshore wave period (s)	12	12, 15, 18
Offshore water depth (m)	21 m	27 m
Storm surge level above MSL (m)	+5 m NAP during 5 h	+5 m NAP during 5 h
Median sediment diameter (mm)	0.225	0.225
Median fall velocity (m/s)	0.0267	0.0267
Water temperature (°C)	10	10
Cross-shore profile (initial)	a) Dune height at +15 m NAP, b) Dune face with slope of 1 to 3 down to a level of +3 m NAP, c) Slope of 1 to 20 between +3 m and 0 m NAP, d) Slope of 1 to 70 between 0 and −3 m NAP, e) Slope of 1 to 180 seaward of −3 m NAP line	a) Dune height at +15 m NAP, b) Dune face with slope of 1 to 3 down to a level of +3 m NAP, c) Slope of 1 to 20 between +3 m and 0 m NAP, d) Slope of 1 to 70 between 0 and −3 m NAP, e) Slope of 1 to 180 seaward of −3 m NAP line

Remark: mean sea level (MSL) is about equal to NAP.

Table 4.2A
Scale test data of Vellinga (1986) based on prototype conditions of Table 4.1B.

Test	d_{50} (μm)	Te (°C)	w_s (m/s)	$H_{s,o}$ (m)	T_p (s)	h_{toe} (m)	n_{d50} (−)	n_h (−)	n_l/n_h (−)
111	225	12.5	0.0276	0.091	1.31	0.461	1	84	3.9
115	225	13	0.0278	0.091	1.31	0.461	1	84	2.71
112	150	12.5	0.016	0.091	1.31	0.461	1.5	84	2.93
116	150	13	0.0161	0.091	1.31	0.461	1.5	84	1.89
113	130	12.5	0.013	0.091	1.31	0.461	1.730769	84	2.09
117	130	13	0.0132	0.091	1.31	0.461	1.730769	84	1.44
114	95	12.5	0.0082	0.091	1.31	0.461	2.368421	84	2
118	95	13	0.0083	0.091	1.31	0.461	2.368421	84	1.18
101	225	15.5	0.0287	0.163	1.76	0.585	1	47	3.5
105	225	15	0.0285	0.163	1.76	0.585	1	47	2.45
102	150	15.5	0.0168	0.163	1.76	0.585	1.5	47	2.44
106	150	15	0.0167	0.163	1.76	0.585	1.5	47	1.79
103	130	15.5	0.0138	0.163	1.76	0.585	1.730769	47	2.02
107	130	15	0.0137	0.163	1.76	0.585	1.730769	47	1.62
104	95	15.5	0.0087	0.163	1.76	0.585	2.368421	47	1.73
108	95	15	0.0086	0.163	1.76	0.585	2.368421	47	1.4
121	225	10.5	0.0269	0.292	2.35	0.806	1	26	3.08
125	225	9.5	0.0265	0.292	2.35	0.806	1	26	1.95
122	150	10.5	0.0154	0.292	2.35	0.806	1.5	26	2.3
126	150	9.5	0.0152	0.292	2.35	0.806	1.5	26	1.48
123	130	10.5	0.0125	0.292	2.35	0.806	1.730769	26	1.62
127	130	9.5	0.0123	0.292	2.35	0.806	1.730769	26	1.1
124	95	10.5	0.0079	0.292	2.35	0.806	2.368421	26	1.32
128	95	9.5	0.00078	0.292	2.35	0.806	2.368421	26	1.04
Dflume	225	7.5	0.027	1.5–1.7[a]	5.4	4.2	1	5	2

Te = water temperature, w_s = fall velocity, $H_{s,o}$ = deep water wave height, T_p = wave period, h_{toe} = water depth at toe of dune, n_{d50} = ratio of sediment diameter in prototype (= 225 μm) and model, n_h = water depth scale, and n_l = length scale.
[a] Actual wave height was 1.7 m rather than 1.5 m (see Table 6.2 of Steetzel, 1993).

scale test results (Deltaflume) are outside the range of the small-scale test results which means that systematic scale errors are involved. The result of the Deltaflume1986 experiment is somewhat too large, as the offshore wave height during this test is somewhat too large (1.8 m instead of the target of 1.5 m). Neglecting this experiment, the dune erosion volume after 5 h of storm in the prototype is about 250 m^3/m based on the small scale test results and about 300 m^3/m based on the Deltaflume2006 result, which is a systematic error of the order of 20%. Averaging all test results, the dune erosion volume after 5 h can be represented as $A_d = 250 \pm 50$ m^3/m. Using this value, the dune erosion volume above storm surge level in the prototype may be under-estimated by about 50 m^3/m or about 15%.

Table 4.2B
Dune erosion volumes of Vellinga (1986) based on prototype conditions of Table 4.1B.

Test	$A_{t=0.3\ h}$ (m^2)	$A_{t=1\ h}$ (m^2)	$A_{t=3\ h}$ (m^2)	$A_{t=6\ h}$ (m^2)	$A_{t=10.5\ h}$ (m^2)	$A_{t=16\ h}$ (m^2)
111	0.0132	0.0148	0.0158	0.015	0.0174	0.0206
115	0.0082	0.0059	0.0051	0.0037	0.0032	0.0036
112	0.0104	0.0162	0.0176	0.0176	0.0249	0.0313
116	0.0056	0.0066	0.0072	0.0095	0.0117	0.0145
113	0.0056	0.0091	0.0095	0.0116	0.017	0.0193
117	0.0047	0.0055	0.0076	0.0101	0.0145	0.0191
114	0.0049	0.0085	0.0148	0.0191	0.032	0.0389
118	0.0025	0.0058	0.0095	0.0165	0.0242	0.0312
101	0.0449	0.051	0.06	0.0651	0.0765	0.0872
105	0.0331	0.0366	0.0395	0.0419	0.0468	0.0523
102	0.0571	0.0636	0.0776	0.0865	0.0975	0.1153
106	0.032	0.0415	0.0478	0.0529	0.0603	0.0753
103	0.0469	0.054	0.0634	0.0754	0.0903	0.1093
107	0.0261	0.0336	0.0396	0.0448	0.0547	0.0703
104	0.0473	0.0646	0.0956	0.13	0.1648	0.2151
108	0.0266	0.0411	0.0552	0.0739	0.0995	0.1314
121	0.1838	0.225	0.2914	0.323	0.3623	0.3916
125	0.1852	0.1107	0.1265	0.1265	0.1265	0.1328
122	0.1751	0.2207	0.267	0.267	0.2688	0.2747
126	0.1015	0.1293	0.1634	0.169	0.1677	0.1728
123	0.1543	0.2345	0.3129	0.3463	0.3624	0.3836
127	0.0779	0.1435	0.1964	0.2253	0.23	0.247
124	0.161	0.2781	0.3891	0.4644	0.5183	0.5369
128	0.1175	0.1898	0.2673	0.3108	0.3943	0.4538
Dflume	3.92	7.08	11.07	13.3	14.8	–

A = Erosion volume per unit width above storm surge level (in m^3/m).

To reduce systematic errors, an alternative approach has been explored by analyzing the dune erosion data sets for various combinations of exponential values of Eqs. (3.12a), (3.12b) and (3.13), as follows:

1. assume a set of exponents of Eqs. (3.12a), (3.12b) and (3.13);
2. compute the distortion scale n_l/n_h and the length scale n_l by using Eq. (3.13);
3. compute the ratio R of distortion scale used in each test and the distortion scale based on scaling law used;
4. select those tests from Table 4.2A which have a ratio of about $R \cong 1$ (in range of 0.7 to 1.3) with $R = (n_l/n_h)_{used}/(n_l/n_h)_{scaling\ law}$;
5. convert the measured erosion volume A_e (above storm surge level) to prototype erosion values using the scale rule $n_A = n_h\, n_{l,scaling\ law}$;
6. convert the time value at which the erosion area has been measured to a prototype time value using: $n_{Tm} = n_h^{\alpha}$ with α in range of 0.5 to 1 (Eq. (3.12b));
7. make a plot of the erosion volume as a function of time (prototype values); if the applied scaling law is perfect, all curves will collapse on one single curve (minimum scatter);
8. repeat procedure for other set of exponents.

Fig. 4.7 shows a plot of the dune erosion area (data of Vellinga, 1986) as a function of time based on:

$$n_l / n_h = (n_{d50})^{-0.5} (n_{s-1})^{-0.5} (n_h)^{0.28} \tag{4.3}$$

$$n_{Tm} = (n_h)^{0.56} \tag{4.4}$$

which produce the best overall results (minimum scatter). Since the data related to the large-scale Deltaflume experiments are relatively large (in upper part of plot), some systematic errors are still present. The dune erosion volume (above SSL) on prototype scale after 5 h is about 300 ± 75 m^3/m. These expressions produce values which are

Table 4.3A
Scale test data of Delft Hydraulics (2004, 2006a,b).

Test	d_{50} (mm)	Te (°C)	w_s (m/s)	$H_{s,o}$ (m)	T (s)	h_{toe} (m)	n_{d50} (−)	n_h (−)	n_l/n_h (−)
Sflume 2004-T03	0.095	9.5	0.006	0.3	1.83	0.7	2.368	30	1.68
Sflume 2004-T01	0.095	9.5	0.006	0.3	2.19	0.7	2.368	30	1.68
Sflume 2004-T02	0.095	9.5	0.006	0.3	2.59	0.7	2.368	30	1.68
Dflume 2006-T01	0.2	8	0.023	1.5	4.9	4.5	1.125	6	2
Dflume 2006-T02	0.2	8	0.023	1.5	6.12	4.5	1.125	6	2
Dflume 2006-T03	0.2	8	0.023	1.5	7.35	4.5	1.125	6	2

Te = water temperature, w_s = fall velocity, $H_{s,o}$ = deep water wave height, T = wave period, h_{toe} = water depth at toe of dune, n_{d50} = ratio of sediment diameter in prototype (= 0.225 mm) and model, n_h = water depth scale, and n_l = length scale.

Table 4.3B
Dune erosion volumes of Delft Hydraulics (2004, 2006a,b).

Test	$A_{t=0.3\ h}$ (m^2)	$A_{t=1\ h}$ (m^2)	$A_{t=2.04\ h}$ (m^2)	$A_{t=3\ h}$ (m^2)	$A_{t=6\ h}$ (m^2)	$A_{t=8\ h}$ (m^2)
Sflume 2004-T03	0.048	0.149		0.246	0.337	
Sflume 2004-T01	0.061	0.171		0.289	0.394	
Sflume 2004-T02	0.061	0.193		0.328	0.419	
Dflume 2006-T01	2.13	4.23	5.88			8.6
Dflume 2006-T02	2.29	4.58	6.32			9.57
Dflume 2006-T03	2.48	5.31	7.1			9.85

A = Erosion volume per unit width storm surge level above storm surge level (in m^3/m).

similar to those of Vellinga (1986), see Eqs. (2.6a), (2.6b), (2.6c), and (2.6d).

Comparing Eqs. (4.4) and (4.2), the morphological time scale is represented by two different expressions. Eq. (4.4) can be used for dune erosion on all scales, whereas Eq. (4.2) is most valid for dune erosion on smaller scales (n_h = 25 to 50). If both beach and dune erosion of a sandy coastal system with sand sizes in the range of 0.2 to 0.5 mm have to be represented correctly in a small-scale physical scale model (with n_h = 25 to 50), Eq. (4.2) should be used.

Fig. 4.8 shows the dune erosion volume in m^3/m on the left vertical scale and relative erosion increase/decrease (A_e = 250 m^3/m at R = 1 is adopted) on the right vertical scale at t = 5 h (all data sets) as a function of the ratio R of the model distortion used and scaling law distortion (based on Eq. (4.3)). The dune erosion volume is determined by using the length scale from the scaling law Eq. (4.3) (black dots) and from the values used in the tests (open dots). The dune erosion volume above SSL increases with R. The fitted curve can be used to correct the dune erosion volume for tests with a ratio R larger than 1.

The scale analysis results on dune erosion show that the scale tests should be done at a distorted scale to properly represent the wave breaking and wave runup processes. Ideally, the distorted scale used in the test should always be the same as the distorted scale according to the scaling law (R = ratio of applied distortion and scaling law distortion = 1). This is not always feasible. For example, the distortion scale of the tests in the Deltaflume is n_l/n_h = 2 to accommodate the reference cross-shore profile in the Deltaflume test. Using Eq. (4.3), the ideal distortion scale is 1.56 resulting in R = 1.28. Hence, the bottom steepness in the Deltaflume is too large compared with the scaling law. Fig. 4.8 based on all test data can be used to correct this effect. For R = 1 the dune erosion volume after 5 h is about 70% of that for R = 1.28.

Fig. 4.9 shows the data of the small-scale and large-scale flume tests performed at Delft Hydraulics (2004, 2006a,b) focussing on the effect of the wave period. The wave period was varied in the range of 12 to 18 s. The offshore wave height was slightly modified to 9 m (compared to 7.6 m used by Vellinga (1986)). The small scale tests had a bottom steepness ratio of R = 1, but the bottom steepness ratio of the Deltaflume tests was R = 1.28 (Deltaflume was not long enough). As a result, the dune erosion volumes obtained in the Deltaflume tests are somewhat too large and have been corrected using a correction factor of 0.65 based on Fig. 4.8.

Fig. 4.9 shows that the dune erosion volume above storm surge level after 5 h increases with increasing wave period (about 18% for T increasing from 12 to 18 s). Scale effects can also be observed as the

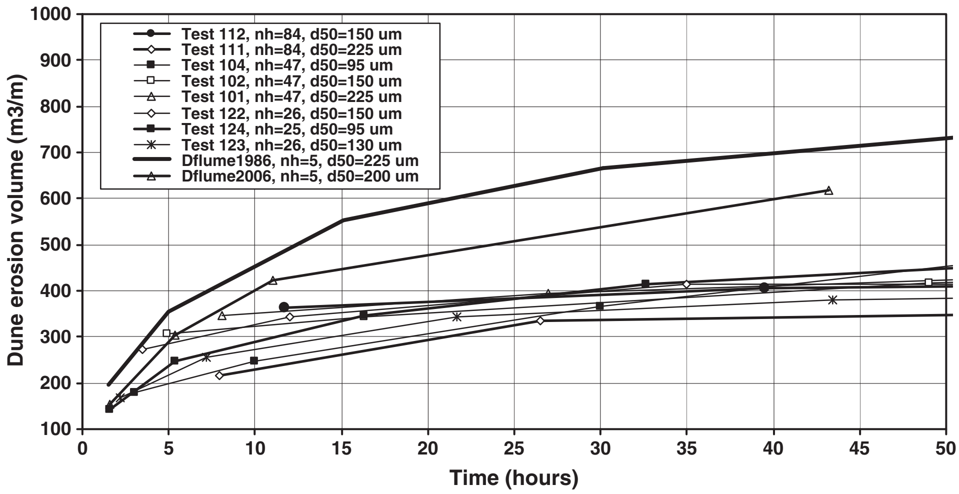

Fig. 4.6. Measured dune erosion volume above storm surge level (SSL) as function of time (prototype values) based on data of Vellinga (1986).

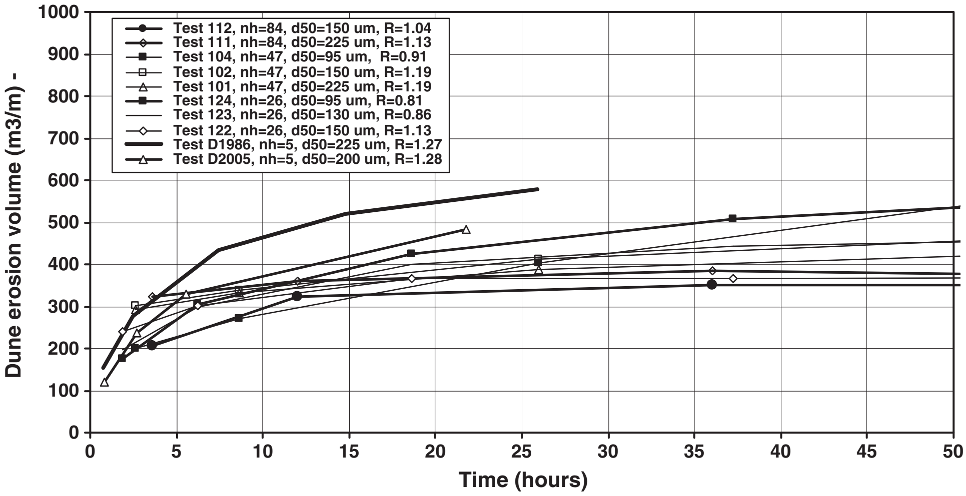

Fig. 4.7. Measured dune erosion volume above storm surge level (SSL) as function of time (prototype values) based on data of Vellinga (1986).

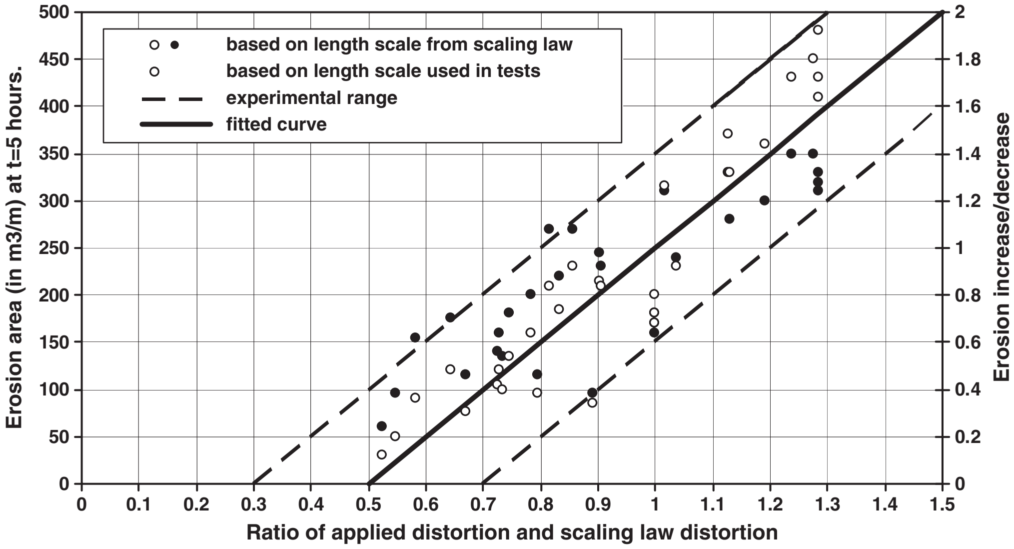

Fig. 4.8. Dune erosion volume (above SSL) at t = 5 h as function of the distortion ratio R (data of Vellinga, 1986).

(corrected) dune erosion volume after 5 h is much larger in the Deltaflume (about 25%) than that in the small scale flume. The scale effects are largest during the first 5 h (at prototype scale) and seem to fade away at larger time scale (10 h). The large scale Deltaflume test of Vellinga (1986) shows slightly larger erosion volumes (about 5% to 10%) after 5 and 10 h than that of Delft Hydraulics (2006a,b).

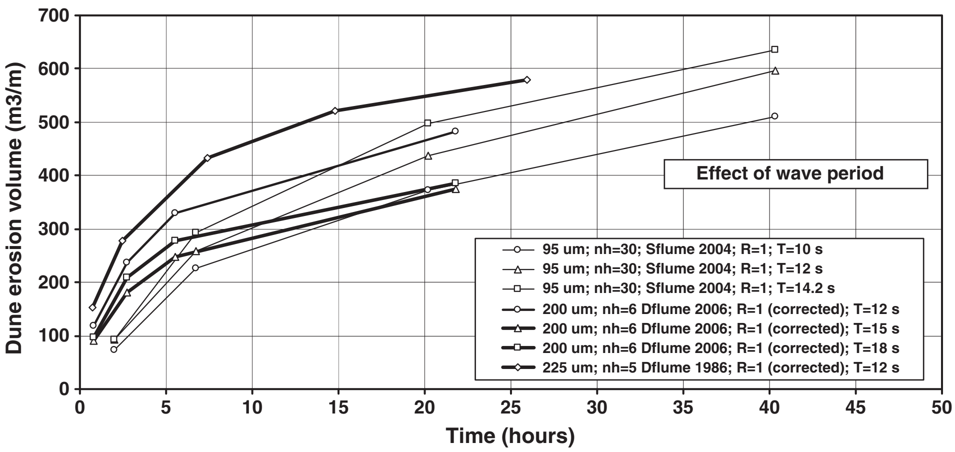

Fig. 4.9. Dune erosion volume (above SSL) as a function of time; Data of tests with scales of 30, 6 and 5 and wave periods between 10 and 18 s (Delft Hydraulics, 2004, 2006a,b).

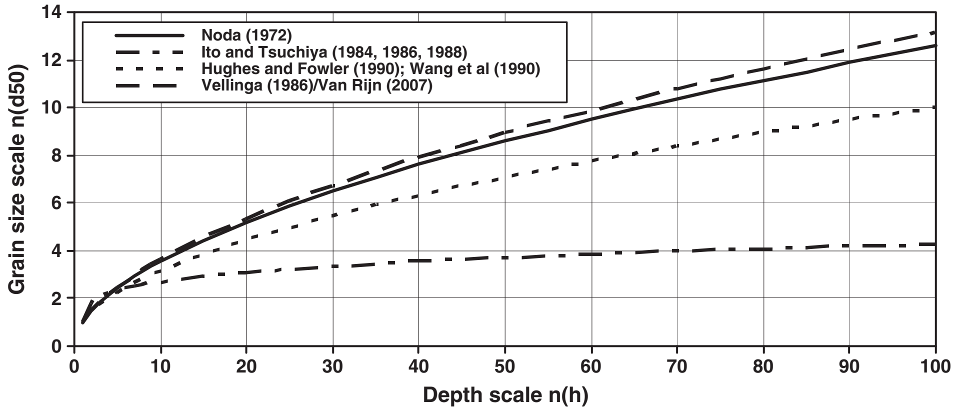

Fig. 4.10. Grain size scale as function of depth scale for various methods.

4.2.3. Discussion of scale-related errors

A first check of the validity of the proposed scaling laws for the beach erosion regime can be obtained by comparing the present results with that from the literature. Assuming fine model sand in the range of 0.1 to 0.5 mm ($n_{d50} \cong n_{ws}$) and undistorted models, the various existing scaling laws are plotted in Fig. 4.10. The grain size scale is plotted as a function of the depth scale. The present scaling law (referred to as Van Rijn, 2007 in Fig. 4.10) reduces to $n_{d50} = (n_h)^{0.56}$ for sand and undistorted scale (see Eq. (4.3)). The scaling laws of Ito and Tsuchiya (1984, 1986, 1988) deviate considerably from the other three scaling laws.

A basic question related to the results of the laboratory dune erosion tests is the applicability of the scale relations to storm conditions in real life (nature). To address this topic, the measured dune erosion volumes along Dutch coast caused by the February 1953 storm can be used. Field data for this event are available for a coastal section (Delfland section; length of about 17 km) between The Hague and Rotterdam. The data comprise cross-shore bed profiles measured a few days after the storm event (post-storm profiles) and bed profiles measured before the storm (pre-storm profiles measured about 3 to 6 months before the storm). The water level during the storm increased from +1.5 m to +3.9 m (above NAP; approx. mean sea level) over a period of about 15 h. The maximum wave height is about $H_{s,o} = 6.3$ m. The local beach grain size is about 0.225 mm. The measured dune erosion volume above the maximum storm surge level of +3.9 m varies in the range of 60 to 150 m^3/m with a mean value of about 90 m^3/m (Vellinga, 1986 and Steetzel, 1993).

Based on the available empirical scaling laws (using depth scale of $n_d = 3.3$ and a length scale of $n_l = 4.6$; $n_l/n_h = 1.4$; $R = 1$), the February 1953 storm including the time-varying storm surge level was simulated by Vellinga (1986) in the Deltaflume of Delft Hydraulics. The dune erosion volume for this (distorted) laboratory test is about 120 m^3/m, which is about 30% larger than the mean observed value of 90 m^3/m for field conditions. These results may indicate that the scaling laws based on (distorted) 2D laboratory tests produce values which are somewhat too large for 3D field conditions. Given the lack of field data for extreme storm conditions, a firm conclusion on the scale errors cannot yet be given.

Finally, it is noted that the present results are only valid for sand material. The influence of the sediment density has not been studied and hence the effect of sediment density in Eq. (4.3) is speculative.

4.3. Application of scaling laws

Basically, the application of scaling laws using sandy materials implies the selection of three parameters: the depth scale (n_d), the length scale (n_l) and the sediment scale (n_{d50}). If the depth scale is different from the length scale, then the scale is distorted.

Usually, the depth scale is a free parameter depending on the available laboratory flume or basin. The sediment scale also is a free parameter, but also depends on the commercially available type of sand. When the depth scale and the sediment scale are known (so two free parameters), the length scale and thus the distortion scale is imposed by Eq. (4.3). The time scale follows from Eq. (4.2) for beach erosion or Eq. (4.4) for dune erosion.

When the length scale or distortion scale is also used as a free parameter (so three free parameters), Eq. (4.3) is violated resulting in scale effects, which can be estimated by using Fig. 4.8. In that case there are two distortion scales (two length scales); the one that is actually used and another one based on Eq. (4.3). The application of the scaling laws will be demonstrated by the following examples:

Method 1 (two free parameters)

1. select depth scale and distortion scale or the sediment grain size scale;
2. find the grain size scale or the distortion scale from Eq. (4.3);
3. find the time scale from Eqs. (4.2) or (4.4);
4. translate the measured results to prototype values using the applied depth and length scale (n_h and n_l).

Example

Given: $n_h = 6$, $n_l/n_h = 1$, $n_l = 6$, $n_{s-1} = 1$ (sand)

Solution: $n_{d50} = 2.7$, $n_{TM} = 2$ based on Eq. (4.2) and $n_{TM} = 2.72$ based on Eq. (4.4) and $n_A = n_h\ n_l = 6 \times 6 = 36$

This latter example represents the experiment with beach slope of 1 to 15 in the small-scale Delft wave flume as the scale model of the Hannover wave flume experiment with beach slope of 1 to 15. According to the scaling laws, the sediment scale should be 2.7 (sediment of 0.1 mm), while a value of 2.1 (sediment of 0.13 mm, see Table 4.1A) has been used given the available sediment material. Hence, the sediment used was somewhat too coarse.

The erosion volume after 6 h in the Delft test is $A_e = 6.5\ (H_{m,o})^2 = 0.18\ m^3/m$, see Fig. 4.5 and Table 4.1A. Using $n_A = 36$ and $n_{TM} = 2$, the predicted erosion volume in the Hannover test is $A_e = 36A_{e,Delft} = 6.5\ m^3/m$ after 2×6 h = 12 h. The measured erosion volume in the Hannover test after 12 h is $A_e = 7\ (H_{m,o})^2 = 6.6\ m^3/m$, see Fig. 4.4, which is in excellent agreement with the value derived from the Delft test result.

Often, Eq. (4.3) cannot be fully matched because of the given flume dimensions (flume not long enough to fit cross-shore profile in flume) and the available sediment material (finer or coarser than required). Then, method 2 based on the input of three free parameters should be used.

Method 2 (three free parameters)

1. select depth scale, distortion scale and sediment size scale;
2. find the time scale from Eqs.(4.2) or (4.4);
3. translate the measured results to prototype values using the applied depth scale (n_h and n_l).
4. compute the ratio of the applied distortion scale and that based on the scaling law and correct the erosion volume based on Fig. 4.8 (use inverse value of the parameter on right vertical scale of Fig. 4.8).

Example

Given: $n_h=6$, $n_l/n_h=1.5$, $n_l=9$, $n_{d50}=2.1$

Solution: n_l/n_h based on scaling law Eq. (4.3) = 1.1, $R=(n_l/n_h)_{used}/(n_l/n_h)_{scaling\ law}=1.5/1.1=1.36$;
correction based on Fig. 4.8 = 1/1.7 = 0.59 and
$n_A=n_h \times n_l \times$
correction coefficient = 6 × 9 × 0.59 = 32
$n_{Tm}=3.6$ based on Eq. (4.2) and $n_{Tm}=2.7$ based on Eq. (4.4).

This example represents the experiment with beach slope of 1 to 10 in the small-scale Delft wave flume (0.13 mm sand) as the distorted scale model of the Hannover wave flume experiment (0.27 mm sand) with beach slope of 1 to 15. The erosion volume after 6 h in the Delft test is $A_e=9\ (H_{m,o})^2=0.25\ m^3/m$, see Fig. 4.5 and Table 4.1A. Using $n_A=32$ and $n_{Tm}=3.6$, the erosion volume in the Hannover test is $A_e=32A_{e,Delft}=8\ m^3/m$ after 3.6 × 6 h = 21.6 h. The measured erosion volume in the Hannover test after 21.6 h is $A_e=9\ (H_{m,o})^2=8.5\ m^3/m$, see Fig. 4.4. Thus, the error is about 10%. This example shows that physical processes in nature can be represented by using a distorted scale model, provided that the scaling laws are properly taken into account.

5. Summary and conclusions

Physical scale models of sandy (quartz) materials have been used frequently to study coastal engineering problems. Scaling laws for coastal movable bed models of are reasonably well established, but the errors due to scale effects are less well understood.

The basic philosophy for movable-bed models is to establish conditions such that all dominant processes are the same in model and prototype. For coastal scale models the most relevant requirement is to attain similarity of the cross-shore equilibrium bed profiles between prototype and model, particularly in the surf zone and the beach and dune zone. In practice, most relevant is the proper representation of the beach and dune erosion volumes. The proper representation of the correct deposition volumes and locations along the cross-shore profile generally is of less importance. This means that the dimensionless parameters describing the equilibrium erosion volumes should be the same in model and prototype. The most dominant mode of transport being either bed load or suspended load (depending on wave conditions and bed material) should be represented correctly. Both undistorted and distorted models have been used in scale modelling.

The ratio of each process parameter in nature (prototype) and in the laboratory model is generally expressed by the scale parameter $n=p_p/p_m$ with p_p = parameter value in prototype and p_m = parameter value in laboratory model. Thus, $n>1$. Correct representation of the physical processes in nature requires that the dimensionless numbers characterizing these processes are the same in nature and in the laboratory model. Examples of these numbers are: the Froude number (subcritical or supercritical flow), the Reynolds number (laminar of turbulent flow), the surf similarity parameter (type of breaking), the Suspension parameter (bed load or suspended load transport), the Shields parameter (intensity of sediment transport and type of bed forms). Often, it is sufficient for correct scale modelling that these dimensionless numbers are in a certain range rather than imposing a fixed value. Analysis of the proper scaling laws shows that the relationship between sediment size and density scale, depth and length scales can be described by Eq. (3.13). The proper values of the exponents can only be determined by calibration using laboratory scale models.

The SANDS EU-Project was focussed on the comparison of a series of laboratory scale experiments using identical wave conditions based on Froude scaling. The experiments were done on three scales: large-scale Hannover wave flume experiment (beach slope of 1 to 15), medium scale Barcelona wave flume experiment (beach slope of 1 to 15) and small-scale Delft wave flume experiments (beach scales of 1 to 10, 15 and 20) using an identical wave train of irregular waves (single topped). The Hannover wave flume is defined to be the prototype. So, the tests in the other wave flumes can be seen as the model test at scale n. The experimental set-up consisted of a horizontal bed followed by a plane sloping beach. The main sediment and hydrodynamic parameters are presented in Table 4.1A.

Since the SANDS data basically represent beach erosion under moderate storm events, another data set representing dune erosion data has been used (see Vellinga, 1986 and Delft Hydraulics, 2004, 2006a,b, 2007). The basic experimental data are given in Tables 4.2A, 4.2B, 4.3A, and 4.3B. The experimental data typically represent beach and dune erosion conditions along the Dutch North Sea coast during super storm conditions.

Both data sets have been used to derive a general set of scaling laws which is valid for both beach and dune erosion under storm conditions, see Eqs. (4.2), (4.3) and (4.4). Eq. (4.2) is most valid for beach erosion, whereas Eq. (4.3) and (4.4) are most valid for dune erosion. The morphological time scale depends on the depth scale and the length scale, and only indirectly on the sediment size scale. The analysis of the scaling laws shows that laboratory scale models can be operated at distorted scales, if necessary.

The present results are in reasonable agreement with those of Vellinga (1986), Hughes and Fowler (1990), Wang et al. (1990). The scaling laws of Ito and Tsuchiya (1984, 1986, 1988) deviate considerably from the other scaling laws.

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