

MODIFIED SEDIMENT PICK-UP FUNCTION
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Abstract

This note addresses the sediment pick-up process in the high-velocity range of 2 to 6 m/s. An existing sediment pick-up function was recalibrated and modified using data of new experiments in a closed pipeline circuit with sand diameters in the range of 50 μm to 560 μm . The new pick-up function was used to simulate the generation and passage of a turbidity current along the submarine Congo canyon offshore the coast of Zaire (Africa).

Introduction

High capacity eroding flows with velocities in the range of 2 to 6 m/s occur during calamity-type erosion events such as: dike and spit breaching; flash floods; turbidity currents, etc. The sediment pick-up during these types of events is largely unknown.

Van Rijn (1984, 1986) has studied the pick-up process of sand particles during low flow conditions with velocities in the range of 0.5 to 1.5 m/s for various types of sand with d_{50} -values in the range of 100 to 1500 μm . Based on experimental data, the following empirical pick-up function was proposed:

$$E = 0.00033 \rho_s [(s-1) g d_{50}]^{0.5} (D^*)^{0.3} [(\theta' - \theta_{cr})/\theta_{cr}]^{1.5} \quad (1)$$

with: E = pick-up rate (in $\text{kg}/\text{m}^2/\text{s}$), $D^* = d_{50} [(s-1)g/v^2]^{1/3}$ = dimensionless grain size parameter (-), d_{50} =median grain size (m), ν = kinematic viscosity coefficient of fluid (m^2/s), $s = \rho_s/\rho_w$ = relative density (-), ρ_s = sediment density (kg/m^3), ρ_w = fluid density (kg/m^3), $\theta' =$ grain-related parameter (-) = $\tau_b'/[(s-1)g d_{50}]$, $\tau_b' =$ grain-related bed-shear stress (N/m^2) = $\rho g [U/C']^2$, U = depth-averaged flow velocity (m/s), R = hydraulic radius (m), (assumed to be equal to water depth), $C' = 5.75 g^{0.5} \log(12R/3d_{90})$ = Chézy-coefficient ($\text{m}^{0.5}/\text{s}$), θ_{cr} = Shields value at initiation of motion (-), g = gravity acceleration (m/s^2).

The amount of grains eroded from the sand bed per unit area and time is defined as the pick-up rate E . Grain movement occurs when the instantaneous fluid force on a grain exceeds the instantaneous resisting force related to the submerged weight and friction force of the embedding of the grain in the sand bed. This can be expressed by the dimensionless Shields parameter (θ) exceeding the threshold value (θ_{cr}). At higher values of the Shields parameter ($\theta \gg \theta_{cr}$), grains are eroded from the sand bed and the top grain layer of the sand bed moves downwards when the suspended concentrations from upstream are low. Equation (1) describing the pick-up rate, is mainly based on the erosion of single grains (grain by grain pick-up) and is only valid for relatively low flow velocities (< 1.5 m/s).

In the high-velocity range (> 1.5 m/s), the behaviour of the sand bed during erosion is different from that in the low velocity range. The pick-up process at high velocities is influenced by the shearing of multiple layers of sand at the top of the sand bed resulting in a slight increase of the porosity (Van Rhee 2010), which is known as dilatancy behaviour in the high-velocity range (> 1.5 m/s). This dilatant behaviour yields an inward hydraulic gradient into the sand bed reducing erosion (dilatancy reduced erosion). Thus, bulk properties such as porosity and permeability are influencing the pick-up process in the high velocity range.

Van Rhee (2010) has extended Equation (1) to the high-velocity regime by incorporating the effect of dilatant behaviour of sand during shearing. This latter effect leads to porewater pressures below the local hydrostatic pressure and thus to an increase of the effective stress between the grains. Consequently, the resisting shear

stress of the sand bed will increase. This effect was incorporated by modifying the critical shear stress for initiation of motion. An adapted critical Shields parameter ($\theta_{cr,d}$) is defined, as follows:

$$\theta_{cr,d} = \theta_{cr} [1 + \{A/(s-1)\} (v_e/k) \{(n_{sl}-n_i)/(1-n_{sl})\}] \quad (2)$$

with: k = permeability of porous sand bed [m/s], n_{sl} = porosity of sheared layer [-], n_i = in-situ porosity of sand [-], v_e = erosion velocity [m/s], A = coefficient [-]. This adapted critical Shields parameter shows that the effect of dilatancy (increase of the in-situ porosity and effect of the hydraulic conductivity) increases the resistance to erosion. When the ratio between the erosion velocity and permeability is low and dilatancy is negligible ($n_{sl} = n_i$), the adapted critical Shields parameter becomes equal to the original critical Shields parameter.

High-velocity experiments in closed pipeline circuit

Experimental setup and instrumentation

To extend the knowledge of sediment pick-up processes in the high-velocity range, a new series of experiments was executed in a closed slurry pipeline circuit of the Dredging Research Laboratory of Delft University of Technology (Bisschop 2018). Various erosion experiments in the high velocity regime were carried out.

The experiment arrangement consisted of a closed circuit in which the sand-water mixture was pumped through a parallel pipeline system (diameter 150 mm) with a measurement section and a by-pass. The measurement section (Figure 1) was rectangular and had an internal height of 288 mm, a width of 88 mm and a length of 6.25 m. The sand-water mixture was pumped through the circuit at a velocity, which was higher than the limit deposit velocity of the mixture. The sand bed in the measurement section was created by lowering the flow velocity in the measurement section by partly closing the valve to the measurement section. The required density of the sand bed was realized by vibrating the measurement section (weak to strong vibration). The by-pass was kept open ensuring that the flow velocity in the whole slurry circuit stayed above the deposition velocity, preventing the circuit from being blocked by deposited sand. The erosion experiment was started by turning the pump at the desired rotation speed and opening the valve to the measurement section to force the eroding flow over the created sand bed, starting the erosion process. The by-pass was kept open during the erosion experiments.

The slurry transport circuit was equipped with the following instruments (Bisschop 2018):

- one radioactive density meter (adjustable in vertical position);
- various conductivity (concentration) probes at different (fixed) levels with vertical spacing of 10 mm;
- pressure gauges measuring the pressure gradient in the measurement section;
- pressure gauges in the vertical loops of the circuit to determine the density of the sand-water mixture;
- pressure gauges to measure pore water pressures in the sand bed;
- two electromagnetic flow meters to measure the total discharge through the slurry circuit and the discharge through the measurement section;
- two-axis electromagnetic flow meter at fixed points in the measurement section (only 2 tests);
- temperature sensor (used to calibrate the conductivity probes).

The density (concentration) of the sand-water mixture and sand bed was determined with the radioactive density meter, which was calibrated in the range of 1000 to 2000 kg/m³ during special tests with known densities. The measured concentrations of this instrument were used to regularly calibrate the conductivity (concentration) probes, which consisted of two transducers (diameter 3 mm) placed 8 mm from each other. Sixteen conductivity probes were placed in the side plate of the measurement section in a sloping matrix distribution with horizontal spacing of 100 mm and vertical spacing of 10 mm, see **Figure 1**. Some lower probes may have been submerged in the sand bed during the test conditions.

The measured potential difference between the transducers depends on the electrical resistance of the medium between these transducers. This is a measure for the density (concentration) of the medium. The electrical resistance also depends on the temperature, the salinity of the water and the type of sand. To determine the influence of the temperature on the electrical resistance, the probes were calibrated for each experiment using the density data of the radioactive density meter. Details on temperature control are given by Bisschop (2018).

The pressure gradient measurements were used to determine the effective bed shear stress during the erosion experiments. The overall density of the sand-water mixture was measured in the vertical loop of the slurry circuit by measuring the pressure gradient in the upward and downward section of the vertical loop. The results of the pressure gradient (dp/dx) measurements along the measurement section were used to determine the frictional characteristics and wall roughness height (k_s) of the measurement section using force equilibrium equation: $A dp/dx + \tau_w W = 0$, with A = area of cross-section above the sand bed (m^2), W = wet perimeter (m), τ_w = wall-shear stress (N/m^2)= $(f_w/8) \rho_m U^2$, U = cross-section averaged flow velocity (m/s); $f_w=8g/C^2$ = Darcy-Weisbach friction coefficient for hydraulic smooth flow (-), ρ_m = density of fluid-sediment mixture (kg/m^3), C = Chézy-coefficient ($m^{0.5}/s$). Based on the measurements with clear water flow, the roughness height of the wall (made of steel with glass windows) of the measurement section was determined to be about $k_s = 0.05$ mm. The area of the cross section above the sand bed and the wet perimeter were not constant during the experiments, because the level of the sand bed decreased as a result of the erosion process. The relative experimental error of the pick-up rate is between 20% for high pick-up rates (in $kg/m^2/s$) and 70% for very low pick-up rates. This error is mainly caused by the fact that the net erosion process is characterized by consecutive erosion and sedimentation rates resulting in irregular changes of the sand bed lever and thus fluctuating pick-up rates. The alternating sequence of erosion and sedimentation processes was caused by vortices close to the bed surface. These vortices consist of sweeps and ejections resulting in downward and upward flow velocities close to the top of the bed. Besides, this error is influenced by the fact that during the erosion process the top of the eroding sand bed was not fully horizontal and the conductivity probes were mounted in a sloping matrix. Another contributor to the error is the uncertainty of the value of the measured near-bed concentration.

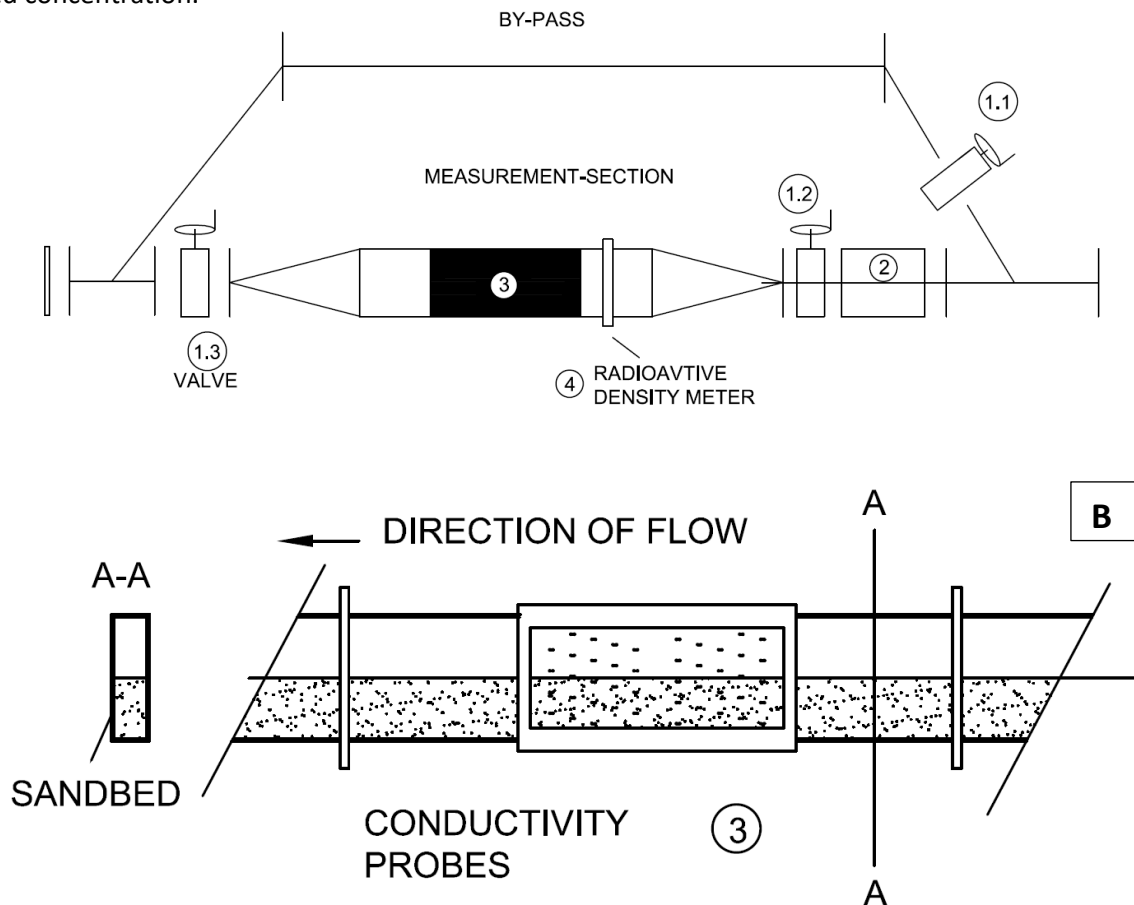


Figure 1 Experimental setup (A) and measurement section (B) of closed pipeline system

Types of sand and experimental program

Four types of sand have been used, see **Table 1**. The very fine Silverbond material consists of fine sand and silt (mainly quartz material). The silt fraction with particles between 2 and 63 μm is about 65%. The permeability of all sand types was determined at different porosities showing that the permeability increases as the porosity increases.

The parameters varied during the experiments were: grain size (type of sand, Table 1), porosity of the sand bed (0.35-0.5), flow velocity (1.5-6 m/s) and density of the eroding flow (1050 to 1400 kg/m^3) upstream of the measurement section. By varying the grain size and bed density, the effect of the permeability of the sand bed on erosion was determined. Relatively low porosity values can be obtained by a combination of deposition procedure and compaction (external vibration by hammering on the steel wall of the measurement section). The minimum and maximum porosity values as given in Table 1 were determined by special tests outside the pipeline circuit using a standard procedure (vibration).

Type of sand	Grain size (μm)			Uniformity coefficient C_u	Porosity n		Permeability k (m^2/s)
	d_{10}	d_{50}	d_{60}		n_{max}	n_{min}	
Silverbond	17	51	57	3.35	0.536	0.385	$1.010^{-5} - 2.0 10^{-5}$
Geba	92	125	133	1.45	0.506	0.370	$8.0 10^{-5} - 1.5 10^{-4}$
Silversand	168	262	285	1.69	0.470	0.370	$3.0 10^{-4} - 7.0 10^{-4}$
Dorsilit	380	562	594	1.56	0.475	0.366	$1.5 10^{-3} - 2.5 10^{-3}$

Table 1 Properties of sand

Measured pick-up rates

Basically, the bed level change (mass change) in time is equal to the pick-up rate (E) minus the sedimentation rate (S) assuming the sand is suspended. The net erosion velocity is determined by the flux of sediment passing a moving interface between an eroding sand bed and eroding flow, see **Figure 2**. The interface between these two zones is the surface of the sand bed and moves downwards with a vertical erosion velocity: $v_e = \Delta z_b / \Delta t$. Net erosion takes place when the pick-up flux exceeds the sedimentation flux ($v_e > 0$). The erosion velocity is negative in the case that the sedimentation flux is exceeding the pick-up flux: settling takes place leading to an increase of the bed height. The eroding sand bed has a porosity n_i , while the eroding flow is considered to have a concentration c_{nb} .

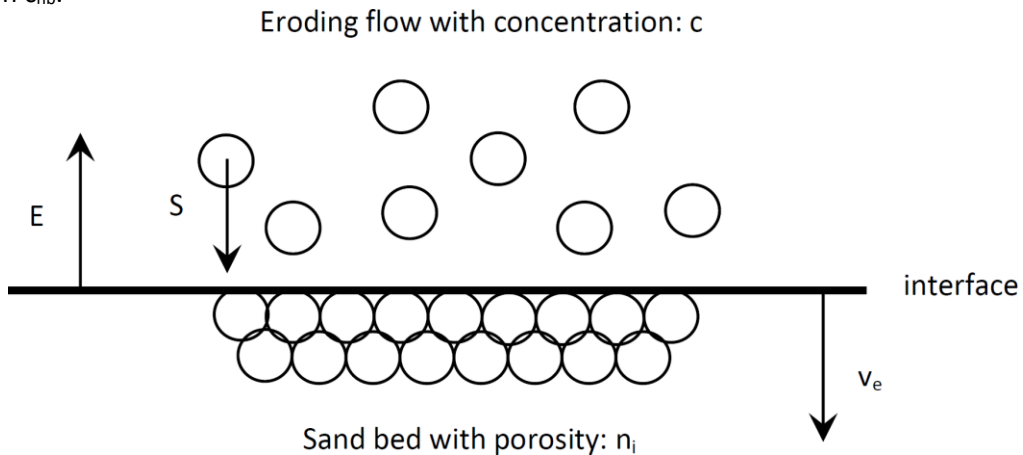


Figure 2 Definition erosion and sedimentation parameters

Defining the relative sedimentation flux (S_{rel}) as the flux relative to the interface between the non-eroded sand bed and eroding flow (Van Rhee 2010, Bisschop, 2018), it follows that:

$$S_{rel} = \rho_s c_{nb} (w_{s,m} - v_e) \quad (3)$$

The flux of grains moving through a downward moving interface is equal to the eroding flux of grains eroding from the sand bed:

$$E - S_{rel} = E - \rho_s c_{nb} (w_{s,m} - v_e) = \rho_s v_e (1 - n_i) \quad (4a)$$

$$E = (1 - n_i - c_{nb}) v_e \rho_s + w_{s,m} c_{nb} \rho_s \quad (4b)$$

with: c_{nb} = near-bed volume concentration derived from conductivity probe data, n_i = in-situ porosity of sand $v_e = \Delta z_b / \Delta t$ = erosion velocity or bed level change derived from conductivity probe data, z_b = top level of sand bed, $w_{s,m} = w_{s,o}(1 - c_{nb})^n$ = hindered settling velocity of near-bed grains in dense mixtures, $w_{s,o}$ = settling velocity of individual grains in clear water (based on Zanke 1977), n = exponent (range of 4 to 5) according to Richardson and Zaki (1954), n_i = porosity value, ρ_s = sediment density ($\cong 2650 \text{ kg/m}^3$). The parameters $v_e = \Delta z_b / \Delta t$ and c_{nb} have been measured during the experiments using four types of sand, see Section 2.

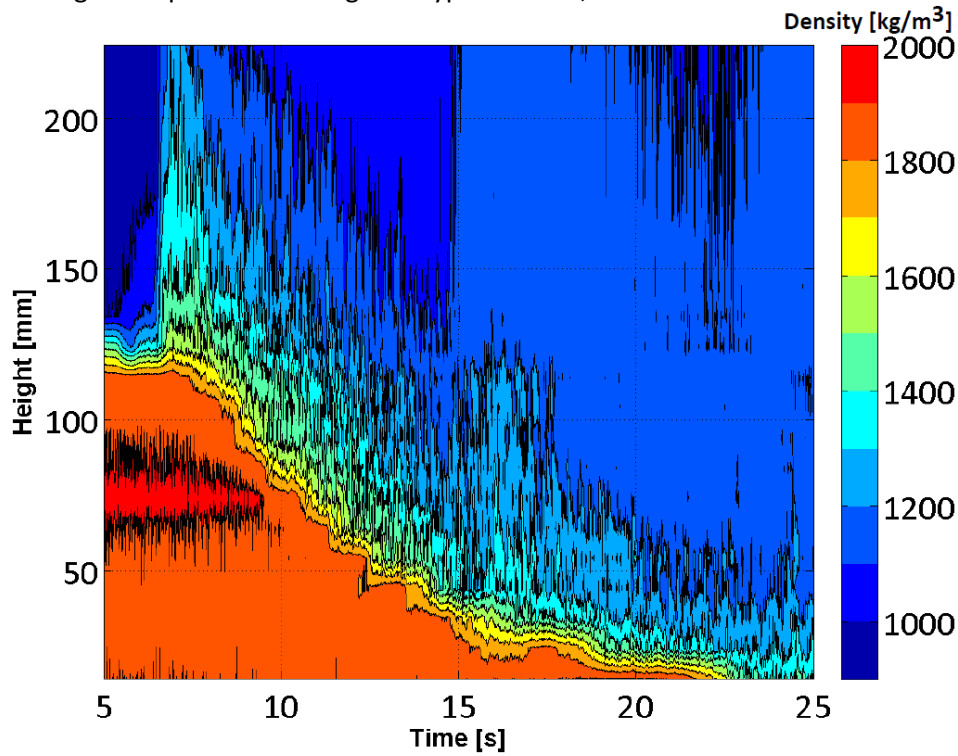


Figure 3 Erosion (height) of sand bed as function of time; density scale (in kg/m^3); sand $262 \mu\text{m}$

A typical example of the erosion velocity (v_e) of the sand bed is shown in **Figure 3**, yielding an erosion velocity of about 6 mm/s for sand of $262 \mu\text{m}$. Equation (4) shows that the pick-up rate (E) depends on the assumed value of the near-bed concentration. The near-bed concentration is not a well defined parameter because the level at which the near-bed concentration should be determined is not well defined. The actual level of the near-bed concentration depends on the concentration profile above the eroding bed and on the dimensions of the turbulent eddies influencing the sedimentation rate by transporting grains from the eroding flow to the eroding sand bed. Visual observations of the erosion process show that these vortices have an average height of approximately 30 mm . Based on this, the near-bed concentration is defined as the average concentration of the

layer with a thickness of 30 mm above the top of the eroding sand bed. Assuming that the scale of the near-bed vortices in the test tunnel will be similar to those in free surface prototype flows, this approach yields sufficiently accurate results for prototype conditions as shown by a sensitivity study (Bisschop 2018).

The trend lines of the measured pick-up rates in the high-velocity range of 1.5 to 6 m/s are shown in **Figure 4**. The trend lines represent the trend of the measured data points, which are scattered around the trend lines. The pick-up rate increases with increasing grain size.

Detailed analysis of the measured data shows the following characteristics (Bisschop 2018):

- the pick-up rate increases at an increasing flow velocity defined as the ratio of the discharge and the area of the cross section above the sand bed;
- the pick-up rate increases at an increasing grain size; at a velocity of 4 m/s the pick-up rate of 562 μm -sand is about 10 times higher than that of 51 μm -sand; fine sand has a very low permeability which may cause relative high porewater underpressures (below local hydrostatic pressure) during shearing, increasing the resistance to shear and hence a reduction of the pick-up rate; the trend of an increasing pick-up rate for increasing grain size was also found for the low velocity range (Van Rijn 1984, 1986);
- the pick-up rate increases at an increasing porosity (decreasing density of the bed); a high porosity value or low density decreases the resistance to shear causing an increase of the pick-up rate;
- the depth-averaged concentration (density) of the eroding flow above the sand bed has almost no effect on the pick-up rate for densities of the eroding flow between 1050 and 1400 kg/m^3 .

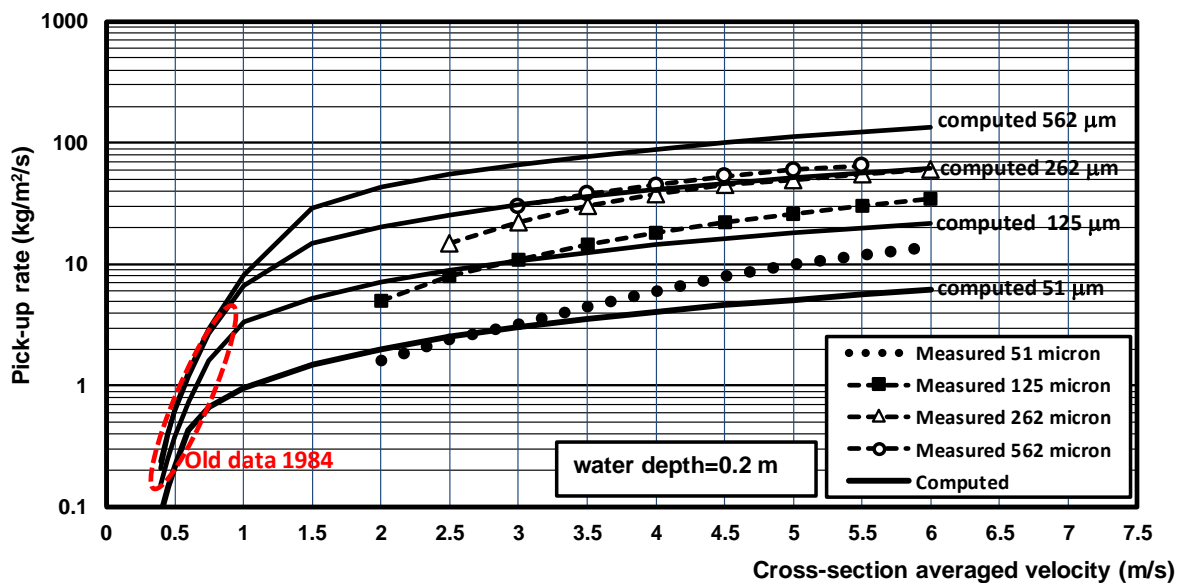


Figure 4 Measured and computed pick-up rates (in $\text{kg}/\text{m}^2/\text{s}$) as function of flow velocity and 4 types of sand

Pick-up-models

Equation (1) which is valid in the low-velocity range only, has been slightly adjusted by introducing a damping factor f_D . The damping factor takes all additional effects occurring in the high velocity range into account. The most important processes are the damping of turbulence (turbulence collapse) in the near bed-area where the sand concentrations are extremely large, the increase of the kinematic viscosity and the dilatant behaviour of the top of the sand bed (apparent increase of shear resistance).

The new pick-up function reads, as:

$$E = \alpha \rho_s [(s-1) g d_{50}]^{0.5} (D^*)^{0.3} f_D [(\theta' - \theta_{cr})/\theta_{cr}]^{1.5} \quad (5)$$

with: $\alpha=0.00033$ ($\pm 30\%$ uncertainty); $f_D=1/\theta'$ for $\theta'>1$.

Figure 4 shows computed results of Equation (5) for four grain sizes. Equation (5) produces the correct trends of increasing pick-up rates at increasing flow velocities and grain sizes. At high velocities the pick-up rate according to Equation (5) is linearly proportional to the flow velocity instead of flow velocity to a power of 3 (for low velocities). This reduction in power is also shown in Winterwerp et al. (1992) and Mastbergen and Van den Berg (2003). Equation (5) is only valid for clean, sufficiently dense packed sand without clay and a porosity in the range of 0.4 ± 0.03 . The measured porosity values vary in a wider range of 0.35 to 0.5. For reasons of simplicity, the f_b -parameter of Equation (5) is only related to the grain-Shields parameter, but in reality it is also related to porosity, permeability and dilatancy parameters of the sediment bed. These latter effects are neglected so far. The effect of dilatant processes during shearing can to some extent be taken into account by modifying the critical Shields parameter (Equation 2, Van Rhee 2010) or by reducing the pickup rate by introducing the dilatancy property of the sand bed (Mastbergen en Van den Berg 2003).

Most computed values of Equation (5) are within a factor of 2 from the measured values with exception of the very fine sand of $51 \mu\text{m}$ at very high flow velocities ($> 4 \text{ m/s}$). Although discrepancies of a factor of 2 are quite acceptable for sediment transport processes, it is clear that the f_b -factor should be improved (future research). A very detailed pick-up function including dilatancy effect has been proposed by Bisschop (2018). This model is based on the shearing of lumps of sand as caused by turbulent normal wall stresses. The thickness of the sheared lumps is determined on the basis of the resisting force of a wedge to a normal force according to the solution of Rankine (Lambe and Whitman 1969). The depth-averaged flow velocity is used for modeling of the normal stress on top of the sand bed. In the resisting force, the effect of dilatancy resulting in pore water underpressures is included. Using this approach, a good description of the influence of the relative density and the angle of internal friction on the pick-up rate is obtained. The detailed model has not yet been implemented and tested in a mathematical model to compute the pick-up rate due to eroding flows (future research) and is therefore not used in the present paper.

Model applications

The new, simple pick-up function (Equation 5) has been used to simulate the high-velocity turbidity current observed in the submarine Congo-canyon at 4000 m water depth off the coast of Zaire (Khripounoff, 2003). More information about turbidity currents in the Congo canyon is provided by Azpiroz et al. 2017. The Congo Canyon (depth up to 1000 m, width up to 10 km) extends over about 760 km from the river mouth to the ocean floor (water depth of 5100 m) downslope from the Congo-Angola continental shelf, see **Figure 5**. Current meters, turbidity meters and sediment traps were deployed on a mooring cable located in the canyon axis at about 260 km from the shelf edge. The instruments recorded the signature of a very extreme event on March 8, 2001.

Sample bottles at 110 to 150 m above the bottom were filled with fine sand (0.15 to 0.2 mm) and debris. The optical turbidity meter at 400 m above the bed did not show any signs of increased turbidity levels. Hence, the turbidity current extends between 150 and 400 m in the vertical direction. At a station outside the canyon, the turbidity current was observed about three days later (by overflow from the canyon); a bottle at 30 m above the bed was filled with brown clay.

The turbidity current simulation model (Van Rijn 2004) of the Congo Canyon consists of about 7500 grid points with increasing step size from 0.1 m at the entrance of the canyon to 250 m at the most offshore station at 640 km. The model used is a 1D simulation model based on two layers with depth-mean variables (turbidity current layer and dilute flow layer above it) and is triggered by a supercritical turbidity current with a thickness of 50 mm and a sand concentration of 200 kg/m^3 . The boundary conditions are: $h_{x=0}=0.05\text{m}$, $c_{x=0}=200 \text{ kg/m}^3$, $\theta=0.06$ implying an initial velocity of about 0.3 m/s. The triggered turbidity current will either die out or self accelerate depending on the local slope, bed friction, ambient water entrainment, and the pick-up rate at the bed. Using sand of $150 \mu\text{m}$, the turbidity current dies out if the coefficient of Equation (5) is $\alpha=0.00033$, but accelerates over a distance of 200 km for a coefficient of $\alpha = 0.0004$. This value is higher than the calibration value $\alpha=0.00033$, but still within the uncertainty range of $\pm 30\%$. However, it is an indication that Equation (5) may underpredict (by about 50%) the erosion rate of $150 \mu\text{m}$ in a turbidity current. **Figure 4** also shows that Equation (5) underpredicts the measured pick-up rates of $125 \mu\text{m}$ for velocities larger than about 4 m/s.

Figure 6 shows the computed velocity (u_2 ; depth-mean over layer h_2), layer thickness (h_2) and sand concentration (c_2 ; depth-mean over layer h_2) for $d_{50}=150 \mu\text{m}$ using a coefficient of $\alpha= 0.0004$ and $\alpha= 0.0008$.

In the model, self-acceleration did not occur for sand of $200 \mu\text{m}$. The simulated turbidity current is supercritical up to 210 km. It is noted that the conditions forcing a hydraulic jump are not included in the model. At the location of 210 km, the layer thickness is about 400 m and the flow velocity is about 10 to 16 m/s. Beyond 210 km the slope angle of the bed is so small that the turbidity current becomes subcritical and the turbidity will gradually die out due to reducing velocities and concentrations. The transition to subcritical flow may be established by an internal hydraulic jump (not included in the model).

The observed velocity in the initial phase (instrument failure after that) of the turbidity current at 260 km from the shelf edge ($x=0$) was about 1.2 m/s at a height of 110 to 150 m above the bed of the canyon. Based on the available data, the layer thickness of the turbidity current is between 150 and 400 m. The model predicts a maximum layer thickness of about 400 m at 200 km, which is of the right order of magnitude compared to the available data, see **Figure 6**. The layer thickness decreases after that due to deposition in subcritical flow. Measured velocities during full development of the turbidity current are not available (failure of instruments).

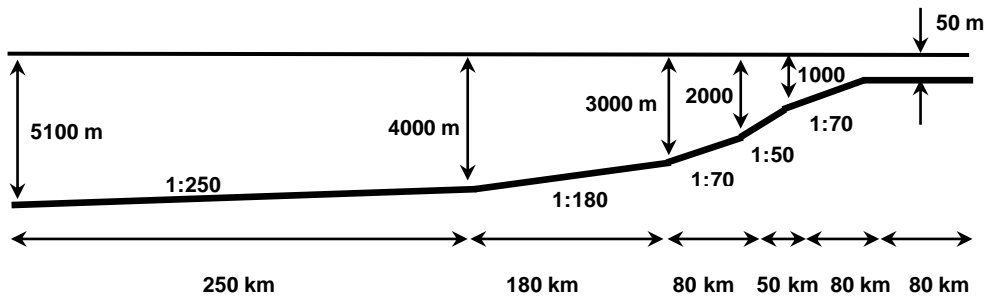


Figure 5 Longitudinal profile of Zaire canyon

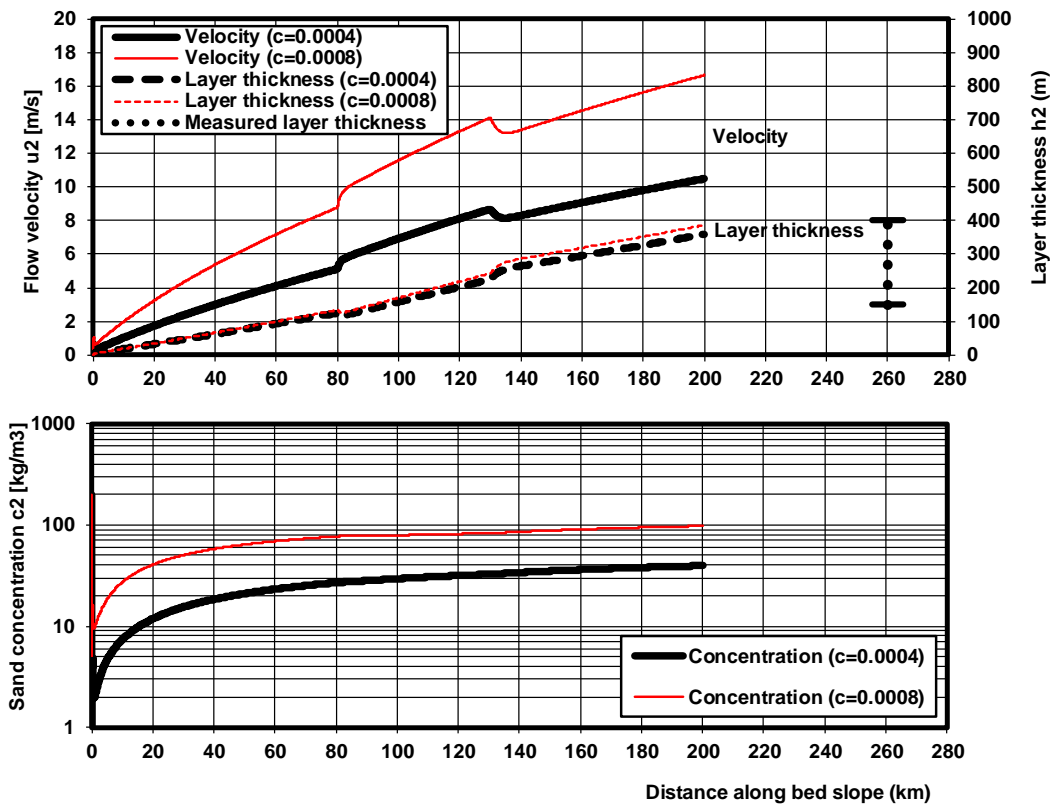


Figure 6 Computed flow velocity, layer thickness and sand concentration of turbidity current, Zaire Canyon

Conclusions

Based on the availability of new data of sediment pick-up rates in the high-velocity range of 1.5 to 6 m/s and sand diameters in the range of 50 to 560 μm , an existing simple sediment pick-up function was recalibrated and applied to simulate the generation and passage of a turbidity current with velocities of about 10 m/s along the Zaire submarine canyon. Based on the turbidity current application with 150 μm sediment, it is found that a fairly simple pick-up function can be used to simulate the generation of a high-velocity turbidity current. Equation (5) is found to underpredict (by about 50%) the pick-up rate of 150 μm sediment. The plot of measured pick-up rates against velocity (Figure 4) also shows that Equation (5) underpredicts (factor 2) the measured pick-up rates of sediment in the range of 50 to 125 μm .

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